

Formuleblad Wiskunde II

(voor studenten Informatica)

- **Exponenten:**

$$(ab)^r = a^r b^r$$

$$a^r a^s = a^{r+s}$$

$$(a^r)^s = a^{rs}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

- **Trigonometrie:**

$$\sin^2 x + \cos^2 x = 1$$

$$\begin{aligned}\cos(2x) &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1\end{aligned}$$

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

- **Limieten:**

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow \infty} e^{-x} = 0$$

$$\lim_{x \rightarrow \infty} \ln x = \infty$$

$$\lim_{x \rightarrow 0+} \ln x = -\infty$$

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x^k} = \infty, \text{ voor vaste } k \in \mathbb{R}$$

$$\lim_{x \rightarrow 0+} x^k \ln x = 0, \text{ voor vaste } k > 0$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^k} = 0, \text{ voor vaste } k > 0$$

- **Rekenregels differentiëren:**

$$\frac{d}{dx} (f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx} (cf(x)) = cf'(x)$$

$$\frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

- **Afgeleiden:**

$$\frac{d}{dx} x^k = kx^{k-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

- Rekenregels integreren:

$$\begin{aligned}
& \int af(x) + bg(x) dx \\
&= a \int f(x) dx + b \int g(x) dx \\
\int f'(g(x))g'(x) dx &= f(g(x)) + C \\
\int f'(x)g(x) dx &= f(x)g(x) - \int f(x)g'(x) dx \\
\int_a^b f'(x) dx &= f(b) - f(a) \\
\frac{d}{dx} \int_a^x f(t) dt &= f(x)
\end{aligned}$$

- Primitieven:

$$\begin{aligned}
\int x^k dx &= \frac{1}{k+1} x^{k+1} + C, \\
&\quad (k \neq -1) \\
\int \frac{1}{x} dx &= \ln|x| + C \\
\int \sin x dx &= -\cos x + C \\
\int \cos x dx &= \sin x + C \\
\int \frac{1}{\cos^2 x} dx &= \tan x + C \\
\int e^x dx &= e^x + C \\
\int \frac{1}{1+x^2} dx &= \arctan x + C \\
\int \frac{1}{\sqrt{1-x^2}} dx &= \arcsin x + C
\end{aligned}$$

- Reeksen:

$$\begin{aligned}
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n, \quad |x| < 1 \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\
\sin(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
\cos(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, \\
&\quad |x| < 1
\end{aligned}$$

- **Complexe getallen:** In het vervolg is $z = x + iy$, $z \in \mathbb{C}$, $x, y \in \mathbb{R}$. Zij $a \in \mathbb{C}$. Dan is $|z - a|$ de afstand van a tot z in het complexe vlak (Gauss-vlak).

$$\begin{aligned}
i^2 &= -1 \\
\bar{z} &= x - iy, \text{ de geconjugeerde van } z \\
|z| &= \sqrt{z \cdot \bar{z}} = \sqrt{x^2 + y^2} \\
|z^n| &= |z|^n \\
\arg(z^n) &= n \arg(z) \pmod{2\pi} \\
e^{i\phi} &= \cos \phi + i \sin \phi \\
e^{\pi i} &= -1
\end{aligned}$$