1. a. Prove that a finite product of discrete spaces is discrete.

   b. For $i \in \mathbb{N}$, let $X_i$ be a discrete space containing more than one point. Prove that $\prod_{i=1}^{\infty} X_i$ is not discrete.

2. a. Prove that a subspace $A$ of a space $X$ is perfect if and only if $A$ is closed and has no isolated points.

   b. In a product space $X \times X$, the set $\{(x, x) : x \in X\}$ is called the *diagonal* of $X \times X$.
   Prove that a space $X$ is Hausdorff if and only if the diagonal of $X \times X$ is a closed set in $X \times X$.

3. Prove that every compact perfect Hausdorff space is uncountable.

4. Let $X$ be a space, $Y$ a Hausdorff space and $f, g: X \to Y$ continuous functions. Prove:

   a. $\{x \in X : f(x) = g(x)\}$ is a closed subset of $X$.

   b. If $f$ and $g$ agree on a dense subset of $X$, then $f = g$.

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