1. Let U be a nonempty open subset of $\mathbb{R}^2$. Prove that U contains a point both coordinates of which are rational.

2. Let X be a set. The *co-countable topology* $\mathcal{T}^*$ on X consists of $\emptyset$ and all subsets O of X with $X \setminus O$ countable.

   a. Prove that $\mathcal{T}^*$ is a topology X.

   b. Let $A \subseteq X$. Prove that the derived set $A' = \emptyset$ if A is countable. Also prove that $A' = X$ if A is uncountable.

   c. Prove that the intersection of an arbitrary countable family of elements $\mathcal{T}^*$ again belongs to $\mathcal{T}^*$.

3. Let X be a Hausdorff space.

   a. Prove that every finite subset F of X is closed in X.

   b. Let $(x_n)_n$ be a sequence in X. Prove that $(x_n)_n$ has at most one limit.

4. a. Let A be a connected subset of a topological space X. Prove that $\overline{A}$ is connected.

   b. Let A be a convex subset of $\mathbb{R}^2$. Prove that $\overline{A}$ is convex.

   c. Give an example of a space X with pathconnected subset A of which its closure $\overline{A}$ is not pathconnected.

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