Tentamen Toegepaste Logica

22 maart 2000

Answers may be given in Dutch or English. Good luck!

Exercise 1. This question concerns minimal logic and simply typed lambda calculus.

a. Give a proof of the formula

$$((D \to B) \to (A \to C)) \to A \to B \to C$$

in minimal logic such that the proof contains a detour and doesn't contain open assumptions.

(5 points)

b. Give the typing derivation corresponding to the proof given as anwer to question 1a.

(5 points)

c. Reduce the term of the answer to question 1b to beta-normal form (it is not necessary to give the typing derivation).

(2 points)

d. Give the proof corresponding to (the typing derivation) of the term found as answer to question 1c.

(5 points)

Exercise 2. This question concerns minimal logic and simply typed lambda calculus.

a. What concept in lambda calculus does a formula correspond to? Explain this correspondence in detail.

(3 points)

b. What concept in lambda calculus does a proof correspond to? Explain this correspondence in detail.

(3 points)

c. What concept in lambda calculus does a detour correspond to? Explain this correspondence in detail.

(3 points)

d. What is provability? To which concept in simply typed lambda calculus does it correspond?

(3 points)

Exercise 3.

a. Suppose that the type `nat` of natural numbers is already defined. Give and explain the definition of an inductive type `natlist` of (finite) lists of natural numbers in Coq.

(5 points)

b. Give the type of the term `natlist_ind` that is automatically generated by Coq once the inductive definition of `natlist` is given.

(5 points)

c. Explain how the term `natlist_ind` is used for induction on lists of natural numbers.

(5 points)

d. Explain the use of the tactic `Inversion`.

(5 points)

Exercise 4.

a. Explain a naive attempt to prove termination of simply typed lambda calculus, and explain why it doesn't work.

(5 points)

b. Give the definition of $X \rightarrow Y$, with X and Y sets of lambda terms.

(3 points)

c. The proof of termination of simply typed lambda calculus as given in the course notes makes use of the *interpretation* of a type A, written as $[A]$. How is the interpretation $[A]$ of a type A defined?

(5 points)

d. Prove, by induction on the typing derivation, that every simply typed lambda term is terminating. Use the result $[A \rightarrow B] = [A] \rightarrow [B]$.

(5 points)

Exercise 5. This question concerns λ-calculus with dependent types (λP).

a. A fragment of λP corresponds to a logic. Which logic?

   (3 points)

b. Give an example of the use of the application rule where dependent types are present.

   (5 points)

c. The typing system for λP contains the conversion rule:

$$\frac{\Gamma \vdash A : B \qquad \Gamma \vdash B' : s}{\Gamma \vdash A : B'}$$

   with $B =_\beta B'$. Explain why this rule is necessary.

   (5 points)

Exercise 6. This question concerns λ-calculus with polymorphic types ($\lambda P2$).

a. The function length takes as input a list of natural numbers, and gives as output a natural number, namely the length of that list.

   We wish to consider a polymorphic version of the length-function, denoted by plength, where the type of the elements of the list is a parameter. What is the type of the function plength?

   (5 points)

b. Let the type of natural numbers be written as Nat and the type of booleans as Bool. Explain how plength can be used to obtain a length-function for lists of natural numbers, and how it can be used to obtain a length-function for lists of booleans.

   (5 points)

Het tentamencijfer is (het totaal aantal punten plus 10) gedeeld door 10.