Opgave 1. (6 + 6)

Prove or disprove CR for the following two TRSs.

(a)

$$\begin{aligned} f(a,x) &\to f(x,x) \\ b &\to a \end{aligned}$$

(b)

$$g(h(g(x))) \to g(h(x))$$

Opgave 2. (4 + 4 + 4 + 4)

Recall that an ARS has the *normal form property* NF if for any normal form n and every a we have $a = n \Rightarrow a \twoheadrightarrow n$. (NB: $=$ stands for conversion.)

Moreover an ARS has the *cofinality property* CF if every a has a (finite or infinite) reduction $a \equiv a_0 \to a_1 \to \cdots$ such that for any b, if $a \twoheadrightarrow b$ then $b \twoheadrightarrow a_n$ for some $n \geq 0$.

Prove or disprove the following implications:

(a) NF $\Rightarrow$ CR

(b) NF $\wedge$ WN $\Rightarrow$ CR

(c) WCR $\wedge$ WN $\Rightarrow$ CR

(d) CF $\Rightarrow$ CR

Opgave 3. (8 + 4)

(a) Prove SN for the TRS:

$$\begin{aligned} f(g(x),y) &\to g(g(f(x,y))) \\ f(x,y) &\to g(x) \end{aligned}$$

(b) Extend the SN-proof in (a) to the TRS with the extra rule

$$f(g(x),y) \to f(x,g(y))$$

Opgave 4. (10 + 4)

(a) Use the Knuth–Bendix method to give a complete TRS for the following specification.

$$\begin{aligned} A(B(x)) &= A(x) \\ B(C(x)) &= B(x) \\ C(A(x)) &= C(x) \end{aligned}$$

TIP: The system can be seen as a string rewriting system by dropping the brackets and the x. This may save writing.

(b) Does the equation $AABCCBBBA = BCBBA$ hold in this specification? And $AABCCBBBA = ABCBBA$ (Motivate the answers without giving actual computations.)

Opgave 5. (4 + 4 + 4 + 4)

Consider the TRS

$$\begin{aligned} c &\to f(g(c)) \\ f(x) &\to x \end{aligned}$$

(a) What is the infinite normal form n of the term c?

(b) Give a transfinite reduction from c to n of length ω.

(c) Give also a transfinite reduction from c to n of length $\omega + \omega$.

(d) Show that with the additional rule

$$g(x) \to x$$

the TRS will not be CR^{∞}.

Opgave 6. (4 + 4 + 4 +4 + 4)

This exercise is about CL.

(a) Give a reduction of the term $SI(KII)(S(II)I)$ to normal form.

(b) Give a complete development of the term $SI(KII)(S(II)I)$.

(c) Give a CL-term W such that $Wy \twoheadrightarrow xyy$.

(d) Consider a fixed-point combinator Y as given. What does it mean that Y is a fixed-point combinator?

(e) Using Y, give a CL-term F such that $F \twoheadrightarrow FF$.

The result is computed as the total number of points plus 10, divided by 10.