TRS

EXAMINATION

3 JUNE 2005

This exam consists of 6 questions. There are 4 pages. Ex. 1 is 13 points, Ex. 3 is 17 points. The other exercises are 15 points each. In total 90 points to be obtained. Ten points are for free. The grade is the total nr of points divided by 10.

Exercise 1. *Only answer in this exercise with yes, no, or don't know. Correct answer is 1 point, wrong answer is -0.5 point, don't know is 0 point. Points to be obtained therefore between -6.5 and 13.*

(1) For all ARSs we have SN & WCR $\Rightarrow$ CR.

(2) For all ARSs we have WN & WCR $\Rightarrow$ CR.

(3) For all TRSs we have: SN $\Rightarrow$ AC.
Here AC is the property 'acyclic', i.e. there are no reduction cycles.

(4) For all TRSs we have: AC $\Rightarrow$ SN.

(5) AC is modular.

(6) CL has the property AC.

(7) S-termen have the property SN.

(8) S-termen have the property AC.

(9) CL has the property (CR^{∞}), infinitary confluence.

(10) CL has the property $UN^{\rightarrow}$.

(11) In lambda-calculus there are 'pure cycles', i.e. terms whose reduction graph consists of a single reduction cycle.

(12) In CL there are 'pure cycles'.

(13) The elementary diagrams belonging to the braids problem, are decreasing.

Exercise 2.

(i) Draw the term formation tree of the lambda term (λy.yyy)((λab.a)I(SS)).
(Here is I ≡ λx.x en S ≡ λabc.ac(bc).)

(ii) Reduce this term to normal form.

(iii) Extend CL with: constants F, A, B and three reduction rules in applicative notation in Table 1. Prove that this extension is again CR. (K ≡ λxy.x .)

1.	FABx → I
2.	FxAB → K
3.	FBxA → S

Table1.

(iv) For just CL alone , $\mathbb{F}_{l.o.}$, the leftmost-outermost reduction strategy, is normalising. Does this still hold for the present extension?

Exercise 3.

(i) Define 'd-rule' (duplicating rule). Likewise for 'c-rule' (collapsing rule).

(ii) What is the theorem of Middeldorp and Rusinowitch?

(iii) An SRS (string rewrite system) corresponds with a TRS. With what TRS rule is the SRS rule10 → 0001 corresponding?

(iv) Are the rules of an SRS (viewed as the corresponding TRS rules) c- or d-rules ?

(v) Prove that for SRSs the property SN is modular.

(vi) Is WN also modular for SRSs?

Exercise 4. Given is the SRS treated in Chapter 1 of the course:

1.	ab → bbba
2.	ba → a
3.	aa → bbbb
4.	bb → b

Table 2.

(i) Prove that this SRS is SN, with the IPO method.
Do this *only* for the 3rd rule(aa → bbbb) and the 4th regel (bb → b).

(ii) Give 2 critical pairs of this SRS. Are they convergent?

(iii) Prove that the SRS is also CR. Mention what lemma's and theorems are used.

Exercise 5.

Given is the TRS with constants A, C, a unary B, and with the two rules as in Table3:

$A \rightarrow B(A)$ $B(x) \rightarrow C$

Table 3.

(i) Indicate with YES/NO whether the following properties hold for this TRS:
WCR, CR, orthogonal, SN, WN, $UN^{\rightarrow}$, NF, AC (acyclic, see definition in Ex. 1),
NE (non-erasing).

(ii) Give the reduction graph $\mathcal{G}(A)$ of the term A, for the case of ordinary, finitary rewriting (so not infinitary rewriting).

(iii) Consider the same TRS, but now for the case of infinitary rewriting. The term A reduces to an infinite term term, namely B^{ω}; what is that reduction and why is it strongly convergent?

(iv) Give the reduction graph of the term A in the case of infinitary rewriting. What length can the reductions in this TRS have?

Exercise 6.

An ARS $\mathcal{A} = \langle A, \rightarrow_1, \rightarrow_2$ has reduction relations $\rightarrow_1$ and $\rightarrow_2$.
They have the following elementary reduction diagrams(see Figure 1):

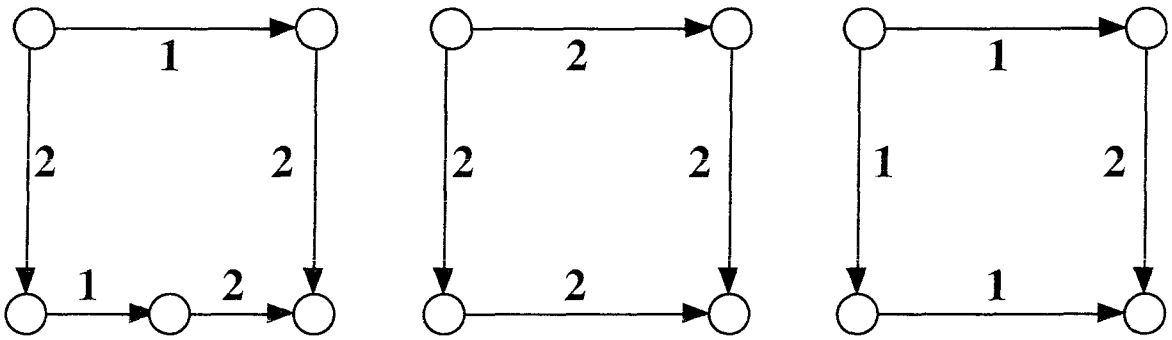

Figure 1.

(i) Is the union $\rightarrow_{12}$ CR?

(ii) (See Figure 2. For which values of x and y, natural numbers 0,1,2,3,..., is this e.d. decreasing, with respect to the usual ordering < on natural numbers?

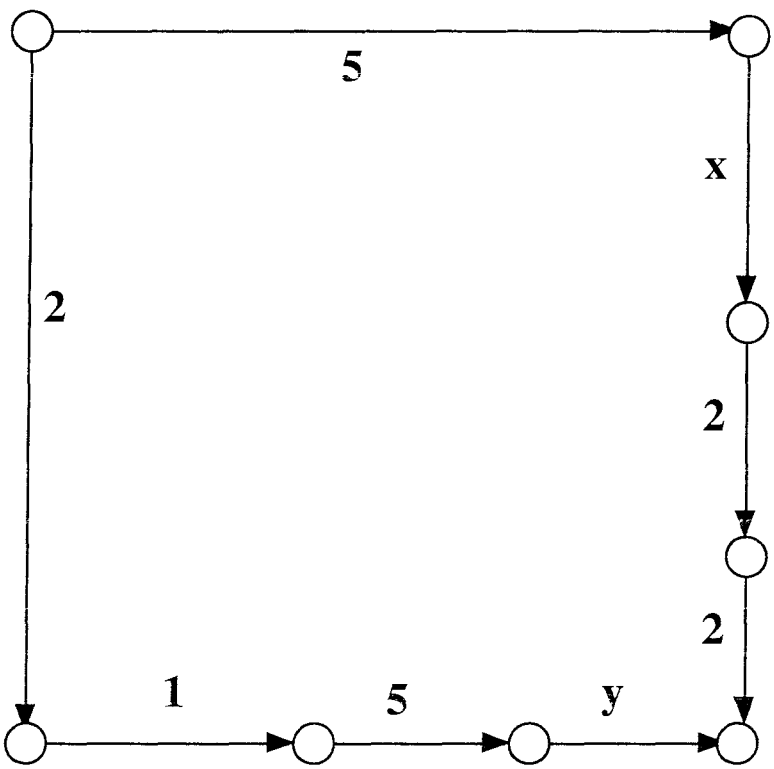

Figure 2.

End. Good luck!