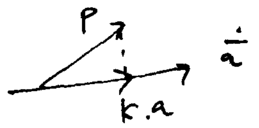
Herkansing 4 juli 2000 Basiswiskunde

1) a) Bereken de inproducten $\bar{a}\cdot\bar{b} = 2-2+0=0$; $\bar{a}\cdot\bar{c} = 2+8-10=0$ en $\bar{b}\cdot\bar{c} = 4-4+0=0$. Omdat de inproducten 0 zijn, staan de vectoren loodrecht op elkaar.

b) Los op: $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 2 & -1 & 4 & 0 \\ 2 & 0 & -5 & 11 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -5 & 0 & -10 \\ 0 & -4 & -9 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & 9 \end{array}\right]$

We vinden $x=3$; $y=2$ en $z=-1$.

c) $\cos\alpha = \dfrac{\bar{a}\cdot\bar{p}}{|\bar{a}|\cdot|\bar{p}|} = \dfrac{5+22}{3\cdot\sqrt{146}} = \dfrac{27}{3\sqrt{146}}$; $\alpha \hat{=} 42°$.

d) 

$\bar{p} - k\cdot\bar{a} \perp \bar{a}$: Dus $k = \dfrac{\bar{a}\cdot\bar{p}}{\bar{a}\cdot\bar{a}} = \dfrac{27}{9} = 3$.

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2) a) $\displaystyle\sum_{n=0}^{\infty} 3\cdot 2^{1-2n} = \sum_{n=0}^{\infty} 3\cdot 2\cdot\left(\tfrac{1}{4}\right)^n = \sum_{n=0}^{\infty} 6\cdot\left(\tfrac{1}{4}\right)^n$

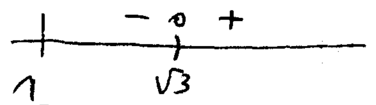
MR met $r=\frac{1}{4}$ heeft som $\dfrac{a}{1-r} = \dfrac{6}{1-\frac{1}{4}} = \frac{4}{3} * 6 = 8$.

b) Dit is een MR met $r = \dfrac{1}{x^2}$. Convergent als $-1 < \dfrac{1}{x^2} < 1$ voor positieve x betekent dat $x > 1$.

De som is dan: $\dfrac{3x}{1-\frac{1}{x^2}} = \dfrac{3x^3}{x^2-1}$

c) Bereken het minimum van $S(x) = \dfrac{3x^3}{x^2-1}$. De afgeleide is $S'(x) = \dfrac{9x^2(x^2-1) - 2x\cdot 3x^3}{(x^2-1)^2} = \dfrac{9x^4 - 9x^2 - 6x^4}{(x^2-1)^2} = \dfrac{3x^4 - 9x^2}{(x^2-1)^2}$

$S'(x) = 0$ voor $3x^2(x^2-3) = 0$; als $x > 1$ (zie boven) is $x = \sqrt{3}$

Tekenverloop $S'(x)$ 

. Dus S is minimaal voor $x = \sqrt{3}$.

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3) a) $f(x) = \sin(2x) - 2\cos x$ dus $f'(x) = 2\cos 2x + 2\sin x$ en $f'(0) = 2$. Raaklijn: $y = f(0) + f'(0)\cdot x$

$y = -2 + 2x$.

b) $\sin(2x) = (2x) - \dfrac{(2x)^3}{3!} + \cdots = 2x - \dfrac{8x^3}{6} + \cdots$

$2\cos x = 2\left[1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \cdots\right] = 2 - x^2 + \dfrac{x^4}{12} - \cdots$

$\sin 2x - 2\cos x = \text{— — .} = \underbrace{-2 + 2x + x^2 - \tfrac{4}{3}x^3 - \tfrac{x^4}{12}}_{\text{vierdegraads polynoom.}} + \cdots$

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4) $P = \dfrac{M}{1+a\cdot e^{-bt}}$; $M = 100$ (eindresultaat!); $P(0) = 20$; dus $a = 4$

$49{,}1 = \dfrac{100}{1+4e^{-9b}} \Rightarrow b = 0{,}15$. $P = \dfrac{100}{1+4\cdot e^{-0{,}15t}}$.

5) a) $\int_1^{e^2} \frac{(x-2)^2}{x}\,dx = \int_1^{e^2} \frac{x^2-4x+4}{x}\,dx = \int_1^{e^2} \left[x - 4 + \frac{4}{x}\right] dx =$

$= \left[\frac{1}{2}x^2 - 4x + 4\ln x\right]_1^{e^2} = \left(\frac{1}{2}e^4 - 4e^2 + 8\right) - \left(\frac{1}{2} - 4 + 0\right) =$

$= \frac{1}{2}e^4 - 4e^2 + 11\frac{1}{2} \approx 9{,}2.$

b) $\int_1^8 x^{1/3}\ln x\,dx = \left[\frac{3}{4}x^{4/3}\ln x\right]_1^8 - \int_1^8 \frac{3}{4}x^{4/3}\cdot\frac{1}{x}\,dx =$

$= \left[\frac{3}{4}\cdot x^{4/3}\ln x - \left(\frac{3}{4}\right)^2\cdot x^{4/3}\right]_1^8 = \frac{3}{4}x^{4/3}\left(\ln x - \frac{3}{4}\right)\Big]_1^8 \approx 16{,}5$

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6) a) grad $f = \begin{bmatrix}3\\-4\end{bmatrix}$; $\tan\varphi = \left\|\binom{3}{-4}\right\| = 5$; dus $\varphi \approx 78{,}7°$

b) grad $f = \begin{bmatrix}3\\-4\end{bmatrix}$ geeft richting van grootste stijging. Loodrecht daarop loopt de niveaulijn. Richting niveaulijn dus $\binom{4}{3}$.

c) $\frac{\text{grad} f \bullet \bar{s}}{\|\bar{s}\|} = \tan\varphi = \frac{\binom{3}{-4}\cdot\binom{1}{1}}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$; Dus: $\varphi \approx -35{,}2°$.

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7) a) $f(x,y) = x^2y + 2xy^2 - 6xy$

$f_x(x,y) = 2xy + 2y^2 - 6y = 2y(x+y-3)$; $f_x(1,1) = -2$

$f_y(x,y) = x^2 + 4xy - 6x = x(x+4y-6)$; $f_y(1,1) = -1$

$f(1,1) = 1 + 2 - 6 = -3.$

Raakvlak: $z = -3 - 2(x-1) - 1(y-1) =$

$= -3 - 2x + 2 - y + 1 = -2x - y.$

b)

	$f_{xx}(x,y) = 2y$	$f_{yy}(x,y) = 4x$	$f_{xy}(x,y) = 2x+4y-6$	Δ	
$(0,0)$	0	0	-6	-36	geen uw
$(0,3)$	6	0	6	-36	geen uw
$(6,0)$	0	24	6	-36	geen uw
$(2,1)$	2	8	2	12	minimum.

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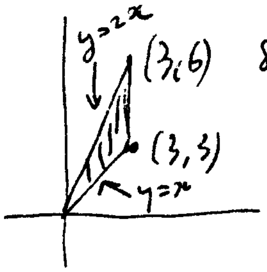
8) a) $\int_1^2 \left(\int_3^4 (x - 2y + 1)\,dx\right) dy = \int_1^2 \left[\frac{1}{2}x^2 - 2xy + x\right]_{x=3}^{x=4} dy = \int_1^2 \left(4\frac{1}{2} - 2y\right) dy =$

$\left[4\frac{1}{2}y - y^2\right]_1^2 = (9-4) - \left(4\frac{1}{2} - 1\right) = 5 - 3\frac{1}{2} = 1\frac{1}{2}.$

b)



Stap1: $\int_x^{2x} \cos(x^2)\,dy = x\cos(x^2)$

Stap2: $\int_0^3 x\cos(x^2)\,dx = \frac{1}{2}\sin(x^2)\Big]_0^3 = \frac{1}{2}\sin 9 \approx 0{,}21$