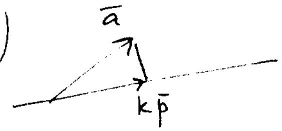
30/3/00; Basiswiskunde voor AL; beknopte uitwerking

1) a) $\left[\begin{array}{ccc|c} 0 & 2 & -3 & 2 \\ 1 & 0 & 2 & -1 \\ 3 & -1 & 0 & 11 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 2 & -3 & 2 \\ 3 & -1 & 0 & 11 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 2 & -3 & 2 \\ 0 & -1 & -6 & 14 \end{array}\right] \sim$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 6 & -14 \\ 0 & 2 & -3 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 6 & -14 \\ 0 & 0 & -15 & 30 \end{array}\right]$; $c_1 = 3$, $c_2 = -2$, $c_3 = -2$; $p = 3a - 2b - 2c$

b) Aan de laatste aangevulde matrix bij a is te zien dat elke rij een spil heeft; dus $\{a, b, c\}$ vormt een basis

c) $\cos\alpha = \dfrac{\bar{b}\cdot\bar{c}}{\|\bar{b}\|\cdot\|\bar{c}\|} = \dfrac{-6}{\sqrt{5}\sqrt{13}} = \dfrac{-6}{\sqrt{65}} \approx -0{,}74$; $\alpha \approx 138{,}1^\circ$.

d) 

$\bar{a} - k\bar{p} \perp p$, dus $\bar{a}\cdot\bar{p} - k\,\bar{p}\cdot\bar{p} = 0$

$k = \dfrac{\bar{a}\cdot\bar{p}}{\bar{p}\cdot\bar{p}} = \dfrac{32}{4+1+121} = \dfrac{32}{126} = \dfrac{16}{63}$.

2) a) $\sum_{n=0}^{\infty} 5\cdot 3^{1-n} = 15 + 5 + \frac{5}{3} + \frac{5}{9} + \dots = \dfrac{15}{1-\frac{1}{3}} = 22\frac{1}{2}$

want MR met $r = \frac{1}{3}$ is sommeerbaar!

b) $r = \frac{1}{x}$ $(x>0)$; $-1 < \frac{1}{x} < 1$ $(x>0)$; dus $x > 1$

De som is $\dfrac{a}{1-r} = \dfrac{5x}{1-\frac{1}{x}} = \dfrac{5x^2}{x-1}$

c) $\dfrac{5x^2}{x-1} = 20$ dus $5x^2 = 20x - 20$ dwz $x^2 - 4x + 4 = 0$; $(x-2)^2 = 0$; $x = 2$.

3) a) $f'(x) = -2\sin(2x) - 2\cos x$; $f'(0) = -2$

$f(0) = 1$; Raaklijn $y = f(0) + f'(0).x = 1 - 2x$.

b) $\cos(2x) = 1 - \dfrac{(2x)^2}{2!} + \dfrac{(2x)^4}{4!} - \dots$

$2\sin x = 2\left(x - \dfrac{x^3}{3!} + \dots\right)$

$\cos 2x - 2\sin x = \underbrace{1 - 2x - 2x^2 + \frac{1}{3}x^3 + \frac{2}{3}x^4}_{\text{4e graads benadering.}} + (\dots)$

4) Stel $y = \dfrac{M}{1+ke^{-st}}$; $y(0) = 25$; $M = \max = 100$ (%); $k = 3$; $y = \dfrac{100}{1+3e^{-0{,}01t}}$

Dus $y = \dfrac{100}{1+3e^{-st}}$ $y(13) = \dfrac{100}{1+3e^{-13s}} = 55$; $s = 0{,}01$

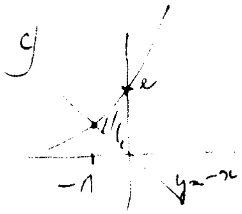
5) a) $\int_1^{e^3} \dfrac{x^2 - 8x + 16}{x^2}\,dx = \int_1^{e^3} (1 - \frac{8}{x} + 16x^{-2})\,dx = \left[x - 8\ln x - \dfrac{16}{x}\right]_1^{e^3} =$

$= \left(e^3 - 24 - \dfrac{16}{e^3}\right) - (1 - 0 - 16) = e^3 - \dfrac{16}{e^3} - 9 \approx 10{,}3$

b) $\int_1^e \underset{f}{x^{2\frac{1}{2}}}\,\underset{g}{\ln x}\,dx = \frac{2}{7}x^{3\frac{1}{2}}\ln x\Big]_1^e - \int_1^e \frac{2}{7}x^{3\frac{1}{2}}\cdot\frac{1}{x}\,dx =$

$= \left[\frac{2}{7}x^{3\frac{1}{2}}\ln x - \left(\frac{2}{7}\right)^2 x^{3\frac{1}{2}}\right]_1^e =$

$= \frac{2}{7}e^{3\frac{1}{2}} - \frac{4}{9}e^{3\frac{1}{2}} - \left(0 - \frac{4}{49}\right) = \frac{10}{49}e^{3\frac{1}{2}} + \frac{4}{49} \approx 6{,}8$

c) 

$\text{Opp} = \int_{-1}^{0} (e^{1+x} - (-x))\,dx = \int_{-1}^{0} (e^{1+x} + x)\,dx = \left[e^{1+x} + \frac{1}{2}x^2\right]_{-1}^{0} =$

$= (e + 0) - (1 + \frac{1}{2}) = e - 1\frac{1}{2} \approx 1{,}2$

6) a) $\frac{\partial z}{\partial x} = 2$ $\frac{\partial z}{\partial y} = -1$; $\text{grad}(z) = \binom{2}{-1}$; $\tan\varphi = |\binom{2}{-1}| = \sqrt{5}$; $\varphi \approx 66^\circ$.

b) Helling in richting $\binom{1}{p}$ is $\dfrac{(1\ \ p)\binom{2}{-1}}{\sqrt{1+p^2}} = \dfrac{2-p}{\sqrt{1+p^2}} = 1$; $p = \frac{3}{4}$

7) $f(x,y) = x^2y + y^2 - 4y$ $f_x(x,y) = 2xy$ $f_y(x,y) = x^2 + 2y - 4$

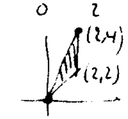
a) $f(0,1) = 0 + 1 - 4 = -3$ $f_x(0,1) = 0$ $f_y(0,1) = 0 + 2 - 4 = -2$

raakvlak $z = -3 + 0 - 2(y-1) = -3 - 2y + 2 = -1 - 2y$.

b) $f_{xx}(x,y) = 2y$; $f_{yy}(x,y) = 2$; $f_{xy}(x,y) = 2x$ $\Delta = f_{xx}f_{yy} - f_{xy}^2$ soort

	f_{xx}	f_{yy}	f_{xy}	Δ	soort
$(-2,0)$	0	2	-4	-12	geen uw
$(2,0)$	0	2	4	-12	geen uw
$(0,2)$	4	2	0	8	minimum

8) a) $\int_0^1 \left(\int_2^3 (2x - y)\,dx\right)dy = \int_0^1 (x^2 - yx)\Big]_{x=2}^{x=3} dy = \dots = 4\frac{1}{2}$

b) 

$\int_1^2 \left(\int_x^{2x} \sin(x^2)\,dy\right)dx = \int_1^2 x\sin(x^2)\,dx = -\frac{1}{2}\cos(x^2)\Big]_1^2 =$

$-\frac{1}{2}\cos 4 + \frac{1}{2}\cos 1$.