Tentamen basiswiskunde AI ; 16-12-1999 (Antwoorden)

1) a) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & -1 & -1 & -2 \\ 3 & 2 & 0 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & -1 & -1 & -2 \\ 0 & 2 & -6 & -20 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -8 & -24 \end{array}\right]$ $c_1 = 2$, $c_2 = -1$, $c_3 = 3$

Dus $\bar{p} = 2\bar{a} - 1.\bar{b} + 3\bar{c}$.

b) $\{\bar{a}, \bar{b}, \bar{c}\}$ is linair onafhankelijk, want bij de eliminatie in a is te zien dat de matrix met $\bar{a}, \bar{b}$ en $\bar{c}$ als kolommen drie spillen heeft. Drie onafh. vectoren in $\mathbb{R}^3$ vormen basis!

c) $\cos(\angle\{\bar{b}, \bar{c}\}) = \dfrac{\bar{b} * \bar{c}}{\|\bar{b}\| . \|\bar{c}\|} = \dfrac{1}{\sqrt{5}\sqrt{5}} = \dfrac{1}{5}$; dus $\angle(\bar{b}, \bar{c}) \approx 78°$.

d) Stel projectie $\bar{q} = \lambda \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ dan geldt $\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} \lambda \\ \lambda \\ \lambda \end{bmatrix} \perp \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

dus $\begin{pmatrix} 0-\lambda \\ -1-\lambda \\ 2-\lambda \end{pmatrix} \perp \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, dwz $-\lambda - 1 - \lambda + 2 - \lambda = -3\lambda + 1 = 0$; $\lambda = \frac{1}{3}$

conclusie $\bar{q} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$.

2) a) $\sum_{n=0}^{\infty} \dfrac{1}{5^{n+1}} = \dfrac{1}{5} + \dfrac{1}{5^2} + \cdots = \dfrac{1/5}{1 - 1/5} = \dfrac{1}{5-1} = \dfrac{1}{4}$

b) Het is een MR met $r = \frac{x}{5}$; convergent als $-1 < \frac{x}{5} < 1$, dus als $-5 < x < 5$

de som is dan $\dfrac{1/5}{1 - x/5} = \dfrac{1}{5-x}$

c) $\dfrac{1}{5-x} = 1$; dus $x = 4$.

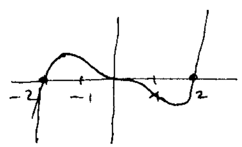
3) a) $\sin x \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$; dus

$\sin 2x \approx 2x - \dfrac{(2x)^3}{3!} + \dfrac{(2x)^5}{5!}$

$2 \sin x \approx 2x - 2 * \dfrac{x^3}{3!} + 2 * \dfrac{x^5}{5!}$

$\sin 2x - 2\sin x \approx -x^3 + \dfrac{1}{4} x^5$.

b)



4) $P(t) = \dfrac{100}{1 + 4e^{-0,2t}}$

5) a) $\displaystyle\int_1^{e^3} \frac{x^2 + 3x - 4}{x^2} dx = \int_1^{e^3} \left(1 + \frac{3}{x} - \frac{4}{x^2}\right) dx = \Big[x + 3\ln|x| + 4/x\Big]_1^{e^3} =$

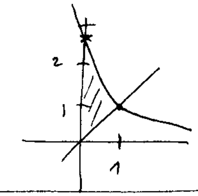
$= \left(e^3 + 9 + \dfrac{4}{e^3}\right) - (1 + 0 + 4) = e^3 + 4 + \dfrac{4}{e^3}$

b) $\displaystyle\int_1^e x^{1\frac{1}{2}} \ln x \, dx = \left[\frac{2}{5} x^{5/2} \ln x\right]_1^e - \int_1^e \frac{2}{5} x^{5/2} \cdot \frac{1}{x} dx =$

$= \left[\dfrac{2}{5} x^{5/2} \ln x - \dfrac{4}{25} x^{5/2}\right]_1^e = \dfrac{2}{5} e^{2\frac{1}{2}} - \dfrac{4}{25} e^{2\frac{1}{2}} + \dfrac{4}{25} =$

$= \dfrac{6}{25} e^{2\frac{1}{2}} + \dfrac{4}{25}$.

5c)



$\displaystyle\int_0^1 (e^{1-x} - x) dx = \left[-e^{1-x} - \tfrac{1}{2}x^2\right]_0^1 =$

$= (-e^0 - \frac{1}{2}) - (-e - 0) = e - 1\frac{1}{2}$.

6) a) Voor elk punt (x, y) geldt $\text{grad}(x, y) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Helling vlak $= \tan\varphi = \left\|\begin{pmatrix} 1 \\ -2 \end{pmatrix}\right\| = \sqrt{5}$; dus $\varphi \approx 66°$.

b) Helling in richting $\begin{pmatrix} 1 \\ p \end{pmatrix}$ is : $\dfrac{(1 \;\; p) \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}}{\sqrt{1+p^2}} = \dfrac{1-2p}{\sqrt{1+p^2}}$.

Deze helling moet 1 zijn. Dus $1 - 2p = \sqrt{1+p^2}$

$1 - 4p + 4p^2 = 1 + p^2$ en $(p \le \frac{1}{2})$

$3p^2 - 4p = 0$

$p(3p - 4) = 0$ $\quad p = 0$ of $\cancel{p = \frac{4}{3}}$ (vervalt)

7) a) $f(x, y) = x^2 - xy^2 + x$; $f(3,2) = 9 - 12 + 3 = 0$

$f_x(x, y) = 2x - y^2 + 1$; $f_x(3,2) = 6 - 4 + 1 = 3$

$f_y(x, y) = -2xy$; $f_y(3,2) = -12$

raakvlak: in $(3, 2, f(3,2))$ $\quad z = 0 + 3(x-3) - 12(y-2)$

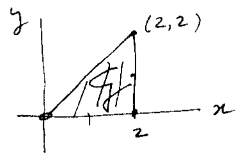
$z = 3x - 12y + 15$.

b)

	$f_{xx}(x,y) = 2$	$f_{yy}(x,y) = -2x$	$f_{xy}(x,y) = -2y$	$\Delta = f_{xx} \cdot f_{yy} - f_{xy}^2$	soort
$(0,1)$	2	0	-2	-4	zadel (geen ext!)
$(0,-1)$	2	0	2	-4	geen ext!
$(-\frac{1}{2}, 0)$	2	1	0	2	minimum

8) a) $\displaystyle\int_1^2 \int_3^4 (x - y) dx\,dy = \int_1^2 \left[\tfrac{1}{2}x^2 - yx\right]_{x=3}^{x=4} dy = \int_1^2 (-y + 3\tfrac{1}{2}) dy = 2$

b)



$\displaystyle\iint_G e^{x^2} dx\,dy = \int_0^2 \int_0^x e^{x^2} dy\,dx =$

$\displaystyle = \int_0^2 x e^{x^2} dx = \frac{e^4}{2} - \frac{1}{2}$