

Basiswissende woerft I 8-4-99

1) a) $\begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 2 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$

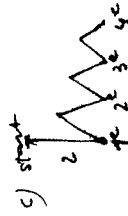
b) $\begin{bmatrix} 1 & -1 & 3 & 8 \\ 1 & 1 & 2 & 4 \\ 1 & -1 & 5 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 3 & 8 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 3 & 8 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$
 $x_3 = 1$
 $x_2 = -1$
 $x_1 = 2$

2) a) $\sqrt{1+9+4+1} = \sqrt{15}$ b) $\cos \alpha = \frac{2 \cdot 6}{\|a\| \cdot \|b\|} = \frac{1+5-4+3}{\sqrt{15} \cdot \sqrt{5}} = 0,2; \alpha \approx 78^\circ$

c) $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 3 & 1 & 1 & 7 \\ -2 & 2 & 1 & -18 \\ 1 & 3 & 1 & -11 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -2 & 0 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -2 & 0 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Dus: $\underline{c} = 49 - 56$

3) a) MR met $r = x^2 - 1$; conv. als $-1 < x^2 - 1 < 1; 0 < x^2 < 2$
 Dus $-\sqrt{2} < x < \sqrt{2}$ en $x \neq 0$.

b) $\text{Sem} = \frac{t_1}{1-r} = \frac{1}{1-x^2-1} = \frac{1}{-x^2} = -\frac{1}{x^2}$; $8-4x^2 = 7; 4x^2 = 1; x = \pm \frac{1}{2}$



weg = $2 + 2 \times 0,8 \times 2 + 2 \times 0,8^2 \times 2 + \dots + 2 \times 0,8^{10} \times 2$
 $= 2 \left[1 + 2 \times 0,8 + 2 \times 0,8^2 + \dots + 2 \times 0,8^{10} \right] =$
 $= 2 \cdot \frac{1 - 0,8^{11}}{1 - 0,8} = 10 (1 - 0,8^{11}) \approx 8,5 \text{ m.}$

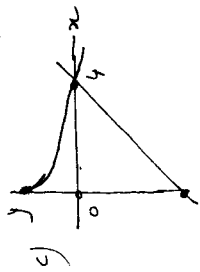
(ii) $2 \times \frac{1}{1-0,8} = 10 \text{ m.}$
 4) a) $a = 5; b = e^{-0,3} \approx 0,74$ b) $N'(t) = 5 \cdot e^{-0,3t} \cdot (-0,3) = -1,5e^{-0,3t}$
 $t = \frac{\ln(-5,97)}{-0,3} \approx -5,97$
 c) $\ln N = \ln 5 - 0,3t; 0,3t = \ln\left(\frac{5}{N}\right); t = 3 \frac{1}{3} \times \ln\left(\frac{5}{N}\right)$

5) a) $\frac{x}{-70} \approx \frac{30}{3+2e^{-0,3t}} < \frac{1}{10}$
 $30 + 20e^{-0,3t} - 30 < \frac{3+2e^{-0,3t}}{10}$
 $198e^{-0,3t} < 3; t > 13,97$

b) $H = \frac{30}{3+2e^{-0,3t}}; 3H + 2He^{-0,3t} = 30$
 $\frac{30-3H}{H} = 2e^{-0,3t}$
 $\ln\left(\frac{30-3H}{H}\right) = \ln 2 - 0,3t$
 Dus: $p = -0,3$
 $q = \ln 2 \approx 0,69$

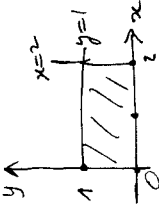
6) a) $\int_4^5 \frac{x^2+x-12}{2x+1} dx = \ln|x^2+x-12| \Big|_4^5 = \ln 18 - \ln 8 = \ln 2 \frac{9}{4} \approx 0,31$
 b) $\int_0^{\frac{\pi}{2}} x \sin 2x dx = -\frac{1}{2}x \cos 2x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\frac{1}{2} \cos 2x dx = -\frac{1}{2}x \cos 2x + \frac{1}{4} \sin 2x \Big|_0^{\frac{\pi}{2}} = -\frac{1}{2}\pi$

c) $\int_0^4 ((2-\sqrt{x}) - (x-4)) dx = \left(6x - \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right) \Big|_0^4 = 10 \frac{2}{3}$



7) $f(x,y) = 2x^2 + 3xy - y^2$
 a) $f_x(x,y) = 4x + 3y$ $f_x(1,2) = 4+6=10$
 $f_y(x,y) = 3x - 2y$ $f_y(1,2) = 3-4=-1$
 grootste stijging in richting gradient: $\begin{pmatrix} 10 \\ -1 \end{pmatrix}$
 b) $z = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$
 $= 4 + 10(x-1) + (-1)(y-2) = 10x - y - 4$
 c) Niveau is 4; richting niveaulijn $\perp$ gradient, dus $\begin{pmatrix} 10 \\ -1 \end{pmatrix}$.

8) a) $\int_0^2 \left(\int_0^1 (y-2x) dy \right) dx =$
 $\int_0^2 \left[\frac{1}{2}y^2 - 2xy \right]_{y=0}^{y=1} dx = \int_0^2 \left(\frac{1}{2} - 2x \right) dx = \left[\frac{1}{2}x - x^2 \right]_0^2 = 1 - 4 - 0 = -3$



b) $\int_1^3 \left(\int_0^x e^{x^2} dy \right) dx =$
 $\int_1^3 e^{x^2} \Big|_{y=0}^{y=x} dx = \int_1^3 x e^{x^2} dx = \left[\frac{1}{2} e^{x^2} \right]_1^3 = \frac{e^9 - e}{4}$

