Basiswiskunde AI toets 3 (30-11-98)

1) a) $F(x)=\int (x+2)^{-\frac{1}{2}}\,dx = 2(x+2)^{\frac{1}{2}}+c = 2\sqrt{x+2}+c$

$F(0)=2\sqrt{2}+c=0$ Dus $c=-2\sqrt{2}$

b) $F(x)=\int (x+\frac{1}{x})\,dx = \frac{1}{2}x^2+\ln|x|+c$

$F(1)=\frac{1}{2}+0+c=3$ dus $c=2\frac{1}{2}$

c) $F(x)=\int \frac{x^2-x-6}{x^2}\,dx = \int (1-\frac{1}{x}-\frac{6}{x^2})\,dx = x-\ln|x|+\frac{6}{x}+c$

$F(1)=1-0+6+c=2$ dus $c=-5$.

2) a) $\int x\sin(x)\,dx = -x\cos x-\int 1.(-\cos x)\,dx = -x\cos x+\sin x+c$

b) $\int x e^{2x}\,dx = \frac{1}{2}xe^{2x}-\int 1.\frac{1}{2}e^{2x}\,dx = \frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+c$

c) $\int x^{-1\frac{1}{2}}\ln x\,dx = -2x^{-\frac{1}{2}}\ln x-\int(-2x^{-\frac{1}{2}})\frac{1}{x}\,dx =$

$= -2x^{-\frac{1}{2}}\ln x-4x^{-\frac{1}{2}}+c = \frac{-2\ln x}{\sqrt{x}}-\frac{4}{\sqrt{x}}+c$

3) a $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \;\frac{u=\sqrt{x}}{du=\frac{dx}{2\sqrt{x}}}\; 2\int e^u\,du = 2e^u+c = 2e^{\sqrt{x}}+c$

b) $\int \frac{x}{(4x^2+7)^6}\,dx \;\frac{u=4x^2+7}{du=8x\,dx}\; \frac{1}{8}\int\frac{du}{u^6} = \frac{1}{8}\int u^{-6}\,du =$

$= \frac{1}{-40}u^{-5}+c$

$= \frac{-1}{40}(4x^2+7)^{-5}+c$

c) $\int \frac{\cos(3x)}{5+2\sin(3x)}\,dx \;\frac{u=5+2\sin(3x)}{du=6\cos(3x)\,dx}$

$= \frac{1}{6}\int\frac{du}{u} = \frac{1}{6}\ln|u|+c = \frac{1}{6}\ln(5+2\sin(3x))+c$

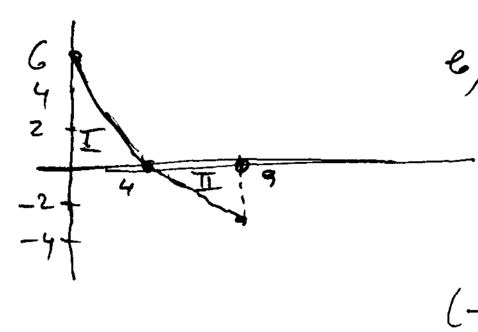
4) a) $f'(x)=\cos(x^2)$ dus $f'(\sqrt{\pi})=\cos(\pi)=-1$

b) $\int_0^{e^2}\ln(x)\,dx = \lim_{a\downarrow 0}\int_a^{e^2}\ln(x)\,dx = \lim_{a\downarrow 0}[x\ln x-x]_a^{e^2} =$

$\lim_{a\downarrow 0}((e^2\ln e^2-e^2)-(a\ln a-a)) = 2e^2-e^2 = e^2$

c) $\int_0^1 \frac{e^{2x}-1}{e^x}\,dx = \int_0^1 (e^x-\frac{1}{e^x})\,dx = \int_0^1 (e^x-e^{-x})\,dx = [e^x+e^{-x}]_0^1 =$

$(e^1+e^{-1})-(e^0+e^0) = e+\frac{1}{e}-2$.

5) 

b) opp I $= \int_0^4 (6-3\sqrt{x})\,dx = [6x-2x^{1\frac{1}{2}}]_0^4 =$

$= (24-2\cdot 4^{1\frac{1}{2}})-0 = 24-2\cdot 8 = 8.$

opp II $= [-6x+2x^{1\frac{1}{2}}]_4^9 =$

$(-54+2\cdot 9^{1\frac{1}{2}})-(-24+2\cdot 4^{1\frac{1}{2}}) = 0-(-8) = 8.$