Exam Applied Stochastic Modeling
12 February 2008

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Question 2 can give 3 points, the other question 2 points.
The use of a calculator is allowed.

1a. Determine the hazard rates of the exponential distribution and the gamma distribution with two phases.
b. Determine for both if the hazard rate is increasing, decreasing, or constant.
c. Let X and Y be i.i.d. exponentially distributed random variables. Calculate $\mathbb{P}(X < t|X+Y > t)$. Relate this to the answer found under a for the gamma distribution.

2. Consider the following generalization of the M/M/1 queue: when a customer arrives and it finds the server occupied, then she leaves the system immediately with probability p instead of joining the queue.
a. Draw the state-transition diagram of this birth-death process.
b. For which values of λ, μ, and p is the model stable?
c. Compute the stationary distribution.
d. What is the distribution of the time that customers spend in the queue?

3. Consider the M/G/1 queue. Let the arrival rate be 1, and let the expected service time be 1/2.
a. Calculate $\mathbb{E}W_Q$ in case the service time has an exponential distribution.
b. Let each service time be exponentially distributed with rate 4 or exponentially distributed with rate 4/3, both with probability 1/2. Verify that $\mathbb{E}S = 1/2$ and calculate again $\mathbb{E}W_Q$.

4. Consider an inventory model with only backorders. Orders arrive according to a Poisson process. The times between replenishments are i.i.d. and have distribution S. Let $\mathbb{E}S = 1$. At the end of each replenishment period all orders are satisfied.
a. Calculate the distribution of the number of outstanding orders at the end of a replenishment period for S constant and exponentially distributed.
b. Calculate the distribution of the time an order has to wait before replenishment for S constant and exponentially distributed.