Exam Stochastic Optimization
10 April 2006, duration: 2 hours

This exam consists of **3** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
The minimal note is 1. All questions are worth 3 points.

1. Consider a Markov chain with state space $\mathcal{X} = \{1,2,3\}$ and transition probabilities $p(1,2) = p(2,3) = p(3,1) = 1$.
a. Does this chain have a limiting distribution, independent of the initial distribution π_0? Motivate your answer.
b. For all possible π_0, give an expression for π_n for all $n > 0$.
c. For which π_0 does the chain have a limiting distribution?
Turn the Markov chain into a semi-Markov process by assuming that $T(x)$ has an exponential distribution with rate x for all $x \in \mathcal{X}$.
d. Give an intuitive explanation for the fact that a limiting distribution exists in this case.
e. Give this distribution.

2. Consider a Markov reward chain with $\mathcal{X} = \{1,2,3\}$, $p(1,2) = p(2,3) = 1$, $p(3,1) = p(3,2) = 1/2$, $r(1) = 1$, and $r(2) = r(3) = 0$.
a. Compute the stationary distribution and use this to compute the average reward.
b. Formulate the Poisson equation and solve it. What is the average reward according to the Poisson equation?
c. Give the definition of the bias and compute it using the answers found under b.

3. Consider a Markov decision chain with $\mathcal{X} = \{1,2,3\}$, $\mathcal{A} = \{1,2\}$, $p(2,a,3) = 1$, $p(3,a,1) = p(3,a,2) = 1/2$ and $r(2,a) = r(3,a) = 0$ for $a = 1,2$, and $p(1,1,2) = 1$, $p(1,2,2) = p(1,2,3) = 1/2$, and $r(1,1) = r(1,2) = 1$.
a. Use policy iteration to find the optimal policy starting with action 1 in all states as initial policy.
b. Which value should $r(1,2)$ have as to make both actions in state 1 optimal?
c. Which policy would then be preferable?