

1. *Dining Philosophers*: Five philosophers sit around a plate of noodles. Between each philosopher there is a single chopstick (so there are five chopsticks in total). A philosopher can only eat if he holds the chopsticks at his left and at his right. When a philosopher is done eating he puts down the chopsticks.
 - (a) Give a μCRL specification of the dining philosophers in which no philosopher is ever permanently excluded from the meal. (You may assume the data type *Bool* and its standard operations as given.) Represent picking up and putting down a pair of chopsticks i and j by actions $u(i, j)$ and $d(i, j)$, respectively.
 - (b) Draw the process graph belonging to your μCRL specification.
2. Let $\text{even} : \text{Nat} \rightarrow \text{Bool}$ be defined such that $\text{even}(n) = \top$ if and only if n is an even number:

$$\begin{aligned} \text{even}(0) &= \top \\ \text{even}(S(n)) &= \neg \text{even}(n) \end{aligned}$$

- (a) Consider the following LPE:

$$\begin{aligned} X(n : \text{Nat}) &= \tau \cdot X(n) \triangleleft \text{even}(n) \triangleright \delta \\ &+ a(n) \cdot X(S(S(n))) \triangleleft \top \triangleright \delta \end{aligned}$$

Prove that the confluence formula for the τ -summand is true, meaning that the τ -transitions generated by this summand are confluent.

- (b) Consider the following LPE:

$$\begin{aligned} X(n : \text{Nat}) &= \tau \cdot X(S(n)) \triangleleft \text{even}(n) \triangleright \delta \\ &+ a(n) \cdot X(S(S(n))) \triangleleft \top \triangleright \delta \end{aligned}$$

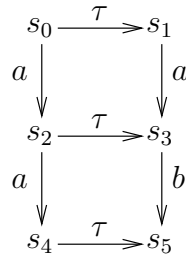
Prove that in this case the confluence formula regarding the τ -summand is false. Also give an example consisting of transitions of $X(0)$ to show that the τ -summand does not give rise to confluent τ -transitions.

- (c) Consider the following LPE:

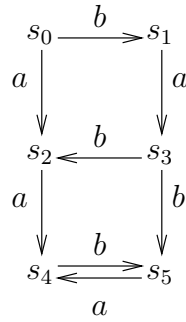
$$\begin{aligned} X(n : \text{Nat}) &= \tau \cdot X(S(n)) \triangleleft \top \triangleright \delta \\ &+ a \cdot X(S(S(n))) \triangleleft \text{even}(n) \triangleright \delta \end{aligned}$$

Prove that in this case the confluence formula regarding the τ -summand is false. Also give an example consisting of transitions of $X(0)$ to show that the τ -summand does not give rise to confluent τ -transitions.

3. (a) Apply the minimisation algorithm modulo branching bisimulation to the process graph below. Describe the subsequent splits that you perform, and the results of those splits.



- (b) What is the maximal collection of confluent τ -transitions in the process graph of (a)? Explain your answer.
- (c) Describe in detail how the model-checking algorithm computes for which states in the process graph below the ACTL formula $E (EX_b \top) U (EX_a EX_b \top)$ is true.



- (d) For each state in the process graph in (c) there exists an ACTL formula that is only true in this state. Give such a characterizing ACTL formula for each of the states.