Exam Probability and Stochastics, May 27, 2008 (12.00-14.45)

It is allowed to use a *basic* calculator, *not* a graphical one. All exercises have equal weight.

1. Suppose that after six exams, your mean score is 8 (an exam score can be any number between 0 and 10 inclusive).
(a) What is the maximal mean score after the next, seventh, exam?
(b) What is the minimal mean score after the next exam?
   Suppose now that I tell you that my mean exam score so far is 9, but I do not tell you how many exams I have had.
(c) If I score a 10 on my next exam, can you compute my new mean exam score? Explain your answer.

2. Consider the following set of values: 3, 3, 5, 7, 9, 9, 10, 15.
(a) Compute the standard deviation of the set.
(b) Give the five-number summary of the set and draw a boxplot.

3. IQ scores are normally distributed with a mean of 100 and a standard deviation of 16. Suppose that many samples of size n are taken from a large population of people, and the mean IQ score is computed for each sample. Describe the mean and standard deviation of the resulting distribution of sample means for $n = 100$ and $n = 400$.

4. Cholesterol levels in men 18 to 24 years of age are normally distributed with a mean of 178 and a standard deviation of 41.
(a) What is the standard score of a 20-year old man with a cholesterol level of 190?
(b) What percentage of men 18 to 24 years of age have a higher cholesterol level?

5. An MP3 player is loaded with 50 musical selctions: 10 rock selections, 15 jazz selections and 25 classical music selections. The player is set on 'random play', so selections are played randomly and can be repeated. What is the probability of each of the following events:
(a) The first four selections are all classical music.
(b) Among the first three selections, none is rock.
(c) The fourth selection is the same genre as the second selection.
(d) The second selection is jazz.

6. We want to estimate a population mean. The sample size is $n = 400$, the sample mean is 140, and the sample standard deviation is 10. Give a 99% confidence interval for the population mean. Explain your method.

7. Give an estimate of the sample size needed to obtain a margin of error of 2 in the 95% confidence interval, if the standard deviation is equal to 10 and we are estimating a population mean.

8. The Bureau of Labour Statistics estimates unemployment rates in the US monthly by surveying 60,000 individuals.
(a) In one month, $3,4\%$ of the 60,000 individuals are found to be unemployed. Find the margin of error for the 95% confidence interval.
(b) If the number of individuals surveyed would be increased to 240,000, by how much would the margin of error change?

9. Consider a test for a population mean μ, where the null-hypothesis is $H_0 : \mu = 25$, and the alternative hypothesis is $H_a : \mu > 25$. The sample gives $\bar{x} = 26$ and the sample standard deviation s is equal to $s = 9$. The sample size is $n = 100$.
(a) Is H_0 rejected when we take significance level 0.05?
(b) Does this change if instead we consider $H_a : \mu \neq 25$? Why?

10. Determine about each statement below whether it is sensible or not. Motivate your answers.
(a) In an estimate for the population mean, in a survey among 400 individuals, the margin of error was 2, and when we extended the survey to 600 individuals, the margin of error became 3.
(b) In a right-tail hypothesis test, the null-hypothesis $H_0 : \mu = \mu_0$ is never rejected at the 0.05 level when the sample mean is below μ_0.
(c) When you reject a hypothesis at the 0.01 level, you also reject it at the 0.05 level.
(d) If you want to reject a hypothesis you have to formulate it as the alternative hypothesis.