Exam Modeling of Business Processes
20 December 2005, duration: 3 hours

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 2, and 4 each give 2 points when correctly answered, question 2 can give 3 points.
The use of a calculator is allowed.
A table of the standard normal distribution is attached.

1. Consider a service center with an infinite number of servers. Customers arrive according to a Poisson process with rate λ. The service time distribution S has distribution function F and expectation β.
a. What is the limiting distribution of the number of customers in the system? And what is its expectation?
b. Give an expression for the expected number of customers that are older than t time units.
c. What is the distribution of the number of customers that are older than t time units? Motivate your answer.

2. Consider a system of two parallel M/D/1 queues. Both have load 80%, but one has service times of length 1 and the other of length 10.
a. Calculate the system times in both systems, and the expected overall system time.
The manager of the system considers merging both queues to obtain economies of scale. We approximate the resulting M/D/2 queue by a single M/D/1 queue with double service speed. Customer are treated in the order of arrival. b. Caracterize the arrival process and the service time distribution of the resulting queueing system.
c. Calculate the system time in this new M/D/1 queue.
d. Compare the results found under a and c and give an intuitive explanation. How would you redesign the system as to obtain the lowest possible average system time?

3. A call center planner uses the Erlang C formula for computing the service level.
a. Give 3 aspects in which the Erlang system does not model most call centers exactly, and explain how this influences the service level.
The planner estimates the input parameters as follows: $\lambda = 10$ and $\beta = 2$. With 24 agents the probability of waiting less than 20 seconds is 0.85, according to the Erlang C formula.
b. What is the productivity?
A colleague analyses the data and says that λ is not always exactly 10, but that it can be somewhere between 9 and 11.
c. How many agents would you schedule to be sure to have approximately an 80% service level? What can you say about the productivity?
d. Explain two possible measures in many call centers that can help to deal with a λ that is not completely known, such that both the service level and the productivity are high.

4. Consider a periodic inventory model where the lead time is equal to the length of the period: an order is placed the moment the previous order arrives. The order policy is as follows: if, after the order arrival, the inventory level is x then an order is placed of $S - x$ items. Unmet demand is backordered. The demand in each period is i.i.d. normal(μ,σ^2) distributed.
a. Give the relation between the order levels of the successive re-order moments.
b. Give a formula for the probability that backorders occur in a certain interval and approximate the average inventory level.
c. Compute these numbers for $S = 10$, $\mu = 4$, and $\sigma^2 = 8$.

Table with value of $P(0<X<x+y)$ with X a random variable with a standard normal distribution

values of x \ values of y	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499