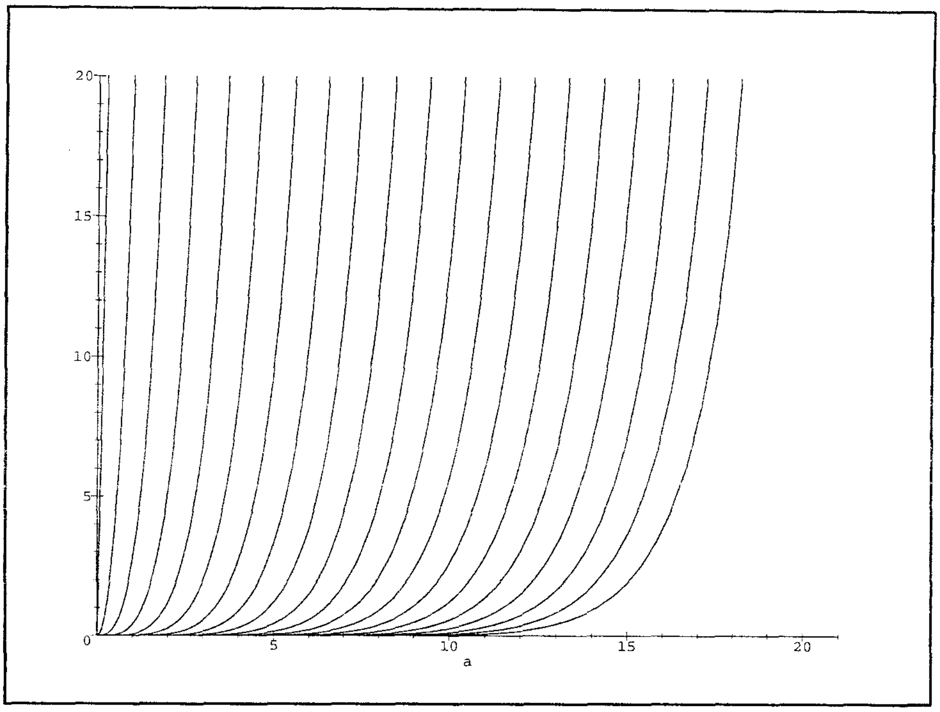
Exam Modeling of Business Processes
20 February 2004

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets each written on one side) with **hand-written** notes.
The minimal note is 1. Question 1 gives 1.5 points when correctly answered, questions 2, 3 and 4 can give 2.5 points.
The answers may be written down in English or in Dutch.
The use of a calculator and a dictionary is allowed.

1. Consider a call center with on average 1.5 arrivals per minute, an average service time of 5 minutes, and 10 agents. The Erlang C model is used to compute the performance of this call center.
a. Compute the expected waiting time using the table on the next page.
b. Give examples of an increase in scope and an increase in scale in the context of call centers.
Suppose the call center doubles in arrival rate and in number of agents.
c. Compute the expected waiting time using the table.
d. How many agents do you need to make the average waiting time less than 90 seconds?



Values of $\mathbb{E}W_Q$ as a function of the load a for (from left to right) $s = 1$ to 20 and $\beta = 60$.

2. Consider a production line with 3 machines and 2 types of jobs. A job of type i visits with probability 0.5 machines i and 3, and with probability 0.5 only machine i. Machine 1 (2) is thus visited by all jobs of type 1 (2), machine 3 is visited by half of all jobs. The arrival processes are Poisson, all service time are i.i.d. exponentially distributed. The average service times of type 1 (2) jobs on both machines they can visit is 1 (2). The arrival rate of type 1 (2) jobs is 0.6 (0.3). The service order at machine 3 is FCFS.

a. Calculate the load of each machine.

b. Calculate the expected waiting times at machines 1 and 2.

c. Describe the arrival process at machine 3. Describe also the service time of an arbitrary job at machine 3.

d. Calculate the expected waiting time at machine 3.

e. Calculate the expected total time that an arbitrary job spends in the system.

3. A small airplane has a capacity of 10 seats. There are 2 fare classes. The price of a type 1 ticket is E 200, of a type 2 ticket E 500. Type 1 customers book before type 2 customers. The demand for type 1 tickets is 10. The demand for type 2 tickets is distributed as follows:

demand	0	1	2	3	4	5	6	7	8	9	10
probability	0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

a. Without overselling, how many seats should you sell to type 1 customers to maximize expected revenue?
b. What is the expected total revenue?
A third type of customers (last minute) is introduced with demand 10, price E 100 per ticket, that book after type 2 customers.
c. What is the expected total revenue when you reserve as many seats as determined under a) for type 2?
d. In this new situation, calculate how many seats should be sold to type 1 customers to maximize expected revenue.

4. Consider a k-out-of-n system with n identical machines. (A k-out-of-n system is up if at least k machines are up.) The time to failure of each machine is exponentially distributed with mean α.
a. Give a formula for the expectation of the time to failure of the system.
b. Give the failure rate of this system for $k = 2$ and $n = 3$.
We add a single repairman to this system, the repair time is exponential with mean 1.
c. Model this system as a birth-death process.
d. For arbitrary n, give an expression for the long-run fraction of time that the system is up.