Exam Modeling of Business Processes
16 December 2003

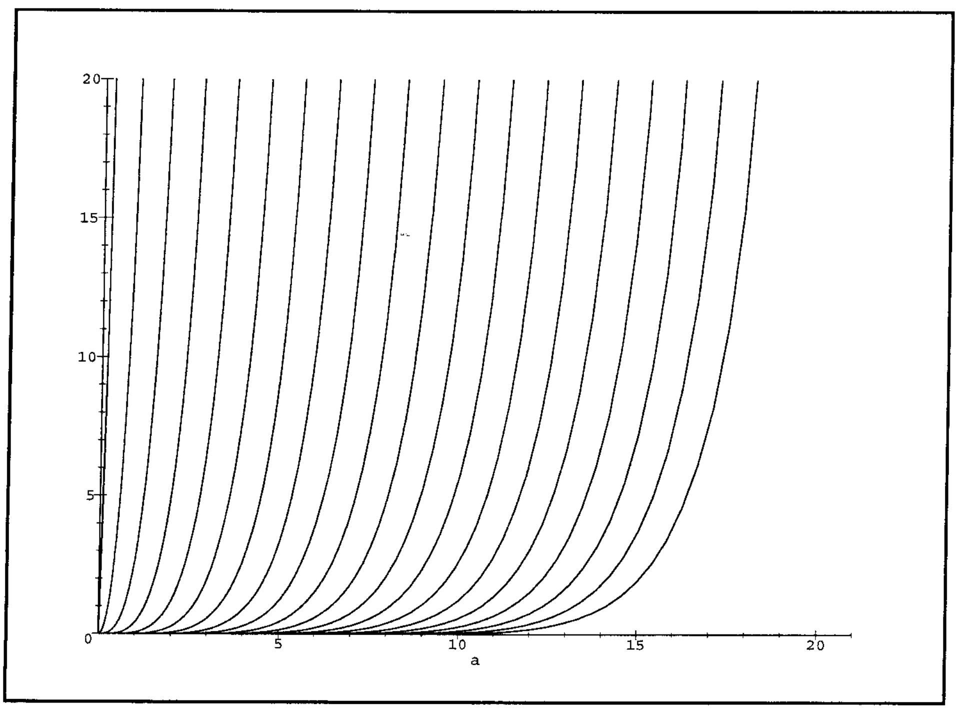
This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets each written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1 and 2 both give 2 points when correctly answered, questions 3 and 4 can give 2.5 points.
The answers may be written down in English or in Dutch.
The use of a calculator and a dictionary is allowed.

1. Consider a system consisting of n identical machines. The time to failure of a machine is exponential with mean 10. The system is up if at least one machine is up. If more than one machine is up, then these spare machines are in cold standby.
a. For $n = 2$, give the probability that the system is up at time 20.
b. What is the failure rate of this system?
We add a single repairman to this system, the repair time is exponential with mean 1.
c. For arbitrary n, give an expression for the long-run fraction of time that the system is up.
d. How many machines are needed to make this fraction at least 0.9999?

2. The objective of aggregate production planning is to find a production plan that minimizes total weighted inventory costs for given order due dates, capacities, and inventory costs.
a. Formulate the linear-programming formulation of aggregate production planning.
If there is no feasible plan (i.e., it cannot be avoided that some orders are late) then there is no feasible solution to the correponding linear problem.
b. Construct an example for which this is the case.
c. Extend the linear-programming formulation to the situation where penalties are paid for every time unit that a job is late.

3. Consider a machine with infinite storage space space to which jobs arrive according to a Poisson process with rate 1. The service time distribution has density $f(x) = 2x$ if $x \in [0, 1]$, 0 otherwise. Jobs are processed in FCFS order.
a. Calculate the first two moments of the service time distribution.
b. Calculate the expected long-run waiting time in this system.
Service times are known upon arrival. It is decided to split the customers in two classes: those with service time below y, and those with service time above y, for some $y \in (0, 1)$. It is decided to give non-preemptive priority to the class with short service times. Within a class the processing order is FCFS.
c. Calculate the first two moments of the service time distributions of both classes.
d. Calculate the expected long-run waiting time in this system.
e. What is the value of y that minimizes the waiting time?

4. Consider a call center with on average 2.5 arrivals per minute, an acceptable average waiting time of 1 minute, and an average service time of 6 minutes. The Erlang C model is used to compute the number of agents.
a. Compute this number using the table.
It is observed that the service time consists of 2 minutes talk time and 4 minutes so-called wrap-up time. Two minutes of this wrap-up time needs to follow the call; the remaining 2 minutes can be done at another time, by another agent. Because of this two agent groups are created: one that handles calls (average service time 4 minutes) and one that does only the second half of the wrap-up (average service time 2 minutes).
b. Compute the number of agents needed in the first group to obtain an average waiting time of less than 1 minute.
c. How many agents are needed in the second group to have a 100% productivity?
Agents like to finish a call completely if possible. It is decided to implement this in the following way. All agents are in 1 group, and they handle a call entirely if the there are few calls waiting. When there are many calls waiting then only the first part of each call is done. The remaining second parts are distributed among free agents later on when the load is lower.
d. How many agents do you expect to need under this new situation? Motivate your answer!



Values of $\mathbb{E}W_Q$ as a function of the load a for (from left to right) $s = 1$ t/ m 20 and $\beta = 60$.