Exam Modeling of Business Processes
10 December 2001

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 2 sheets of paper (or 4 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 2, and 4 each give 2 points when correctly answered, question 3 can give 3 points.
The answers may be written down in English or in Dutch.

1. An Erlang calculator can calculate the service level, defined as the percentage of callers that waits longer than t seconds, based on the arrival rate λ, the average service time β, and the number of servers s.
In a call center there are on average 10 calls per minute, that require each on average 3 minutes to answer. The acceptable waiting time is 20 seconds, and the time between the moment a call is assigned to an agent and the moment it is answered by the agent is around 3 seconds.
a. Give the parameter values for the Erlang calculator by which you can calculate the service level in the call center.
b. To obtain a service level of around 80% 35 agents are needed. Define productivity and calculate it.
c. A model is not an exact description of reality. Give 3 aspects in which the Erlang system does not model the call center exactly.
d. The arrival rate doubles to 20. The manager decides to double the number of agents. What do you expect to be the consequences for the costs and the service level? Motivate your answer!
e. Estimate without using the Erlang formula how many agents need to be scheduled to obtain a 80% service level. Motivate your answer.

2. We consider the availability of a system with 2 components, named A and B, in series. There are two spare components C and D. Component B can be replaced by component D. Component A can be replaced by component C, but only if D replaces B as well. Machines fail independently, whether they are used or not.
a. What are the minimal path sets of this system?
b. Give the functions ϕ and Φ (giving expressions for the availability in a deterministic and in a random environment).
c. If all components have an exponential time to failure with rate 1, and they are all up, what is the expected time to failure of the system?

3. A shop sells goods. When ordered at the beginning of day n, the ordered goods arrive at the beginning of day $n+1$, and they can be sold from that day on. A unit of goods costs p, and is sold for r ($r > p$). Every night that a unit spends in stock costs h. There are no order costs, orders are therefore placed every day.
a. Model this as an inventory model, by introducing random variables for demand, stock, and order sizes. Give the relations between the variables.
b. Express the expected sales at day $n+1$ and the expected inventory costs at the end of day $n+1$ as a function of the stock at the beginning of day n (including the arriving order) and the order placed at the beginning of day n.
c. Suppose that sales on every day are uniformly distributed on $[0, 1]$. Calculate the expected sales at day $n+1$ and the inventory costs of the stock at the end of day $n+1$, given the stock at the beginning of day n (including the order that just arrived), and the order placed at the beginning of day n.
d. Let R be the order policy that miximizes for each day n expected profit minus inventory costs at day $n+1$. Do you think that this order policy maximizes the average expected profit minus inventory costs? Motivate your answer!
The same shop also sells perishable goods. When ordered at the beginning of day n, the ordered goods arrive at the beginning of day $n+1$, and they can be sold during days $n+1$ and $n+2$. After that they are thrown away, without cost nor reward. A unit of goods costs p, and is sold for r. There are no order costs.
e. Model this as an inventory model, by introducing random variables for demand, stock, and order sizes. Give the relations between the variables.

4. A production system consists of 2 production steps. Both take an exponentially distributed amount of time with parameter μ. Production times are independent. Orders arrive according to a Poisson(λ) process. Two different configurations for the system are considered.
a. In the first configuration the two production steps are executed consecutively. There is a large buffer space in front of each production step. Calculate the maximal production rate and the system time as a function of λ.
b. In the second configuration both production steps are executed at the same time. Production on a new order can only start if both steps are finished. There is a large buffer space in front of the combined production step. Calculate the maximal production rate and the system time as a function of λ.
c. Which system has the lowest system time for λ small? Explain this.