Exam Modeling of Business Processes
17th April 2001, 13.30-16.30, Q1.05

This exam consists of **5** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.

1. A consultant is supposed to build a DSS for agent rostering in a call center. She is considering determining the required occupancies using the Erlang formula, and then using a LP approach to find schedules for the agents.
a. Give the definition of a DSS.
b. Formulate in general terms the steps of the modeling process.
c. Give a description of the desired functionality of the DSS of the call center.
d. Describe for the call center case the steps of the modeling process.

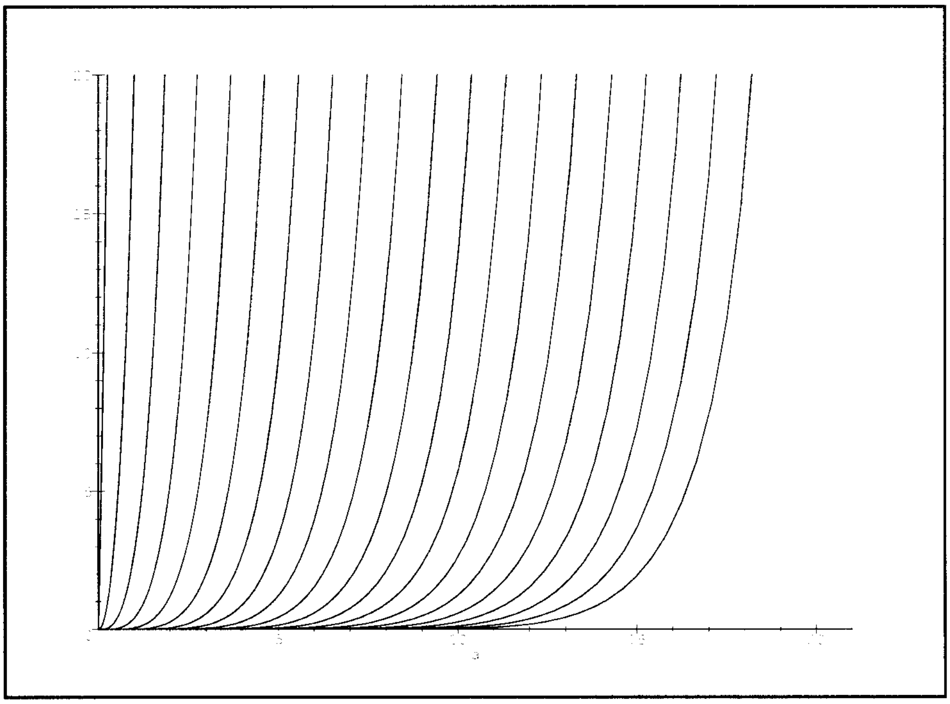
2. A printer can be modeled as an $M/G/1$ queue. For a specific printer it was determined that the arrival rate was 1 and that the average service time was 0.5, and that the printing time of an arbitrary document has approximately an exponential distribution.
a. Calculate the expected time between the moment a printer job is submitted and the moment the printer finishes printing it (the "system time").
It is found that the time to print a job is much longer in reality. The answer lies in printer failures: about 1% of the jobs cause the printer to get jammed. Repair takes on average 30 time units. By lack of data the repair time is assumed to be exponentially distributed. For the sake of the calculation the repair time is added to the printing time.
b. Calculate the first and second moment of this new printing time.
c. Calculate the expected system time if repairs are included.
Hint: the following formula (in the notation of the lecture notes) might be useful:

$$\mathbb{E}W_Q = \frac{\lambda \mathbb{E}S^2}{2(1-\lambda \mathbb{E}S)}.$$

3. A perishable good has order price c. It is sold on day 1 for a selling price r_1. Any remaining goods are sold on day 2 for a discount price of $r_2 < r_1$. Remaining goods after day 2 have no value. Let the demand on day 1 be uniformly distributed on $[0, a]$, and on day two uniformly distributed on $[0, b]$. An order can only be placed at the beginning of day 1.
a. Calculate the total expected demand on day 1 and 2 together.
b. For order size S, calculate the total expected sales on day 1 and 2 together.
c. Give a formula for the expected costs if the order size is S.
d. What is the optimal value of S, if $a = b = 1$?

4. A small town has a single ambulance for dealing with all emergencies in the area (24 hours a day, 7 days a week). Research shows that emergency calls arrive according to a Poisson process. Emergency handling time consists roughly of driving time to the site of the accident, handling time on the site, and driving time to the hospital.
a. Define the variables involved in this system.
b. Give a formula for the probability that the ambulance is busy at the moment an emergency call arrives. Did you make any assumptions?
c. Derive an approximation for the expected time between an emergency call and the moment the ambulance arrives at the site of the accident. Did you make any (additional) assumptions?
d. Indicate how you could answer questions b and c for the case of 2 ambulances.

5. A call center has 12 agents, calls arrive at a rate of on average 15 per minute, and the average call duration is 25 seconds.
a. Calculate the expected waiting time using the table.
It was found that this does not match with reality. Further research showed that it takes on average 5 seconds before an agent pick up the phone after a call is assigned to an agent.
b. Calculate again the expected waiting time of an arbitrary call.
Again, there is a considerable difference between reality and the prediction of the model. Further research showed that agents take short breaks, totaling on average 5 minutes per hour.
c. Give an approximation for the expected waiting for this new situation.
Further research showed that calls abandon at an exponential rate.
d. Extend the $M/M/s$ model to include the abandonments.
e. Give a formula for the expected waiting time in the case of abandonments.



Values of $\mathbb{E}W$ as a function of the load a for (from left to right) $s = 1$ to 20 and average service time 60.

Do not forget to motivate **all** answers.
All problems have equal weight in the final note.