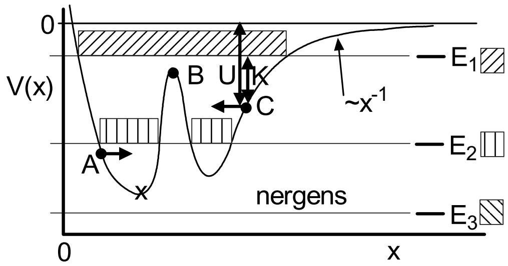
Mechanica deeltentamen 1 - 20 oktober 2003

1. wiskunde
   a. $\sqrt{\frac{1}{1+x}} \simeq \sqrt{1-x} \simeq 1 - \frac{1}{2}x$ dus $a = -\frac{1}{2}$
   b. $e^{ai+b} = e^b \cos a + ie^b \sin a$ dus $p = ie^b$ en $q = a$ en $r = e^b$ en $s = a$
   c. $\sin x = x - \frac{1}{6}x^3$
   d. KV: $a\lambda^2 + b^2\lambda = 0$ Alg Opl met $\lambda_1 = 0$ en $\lambda_2 = -\frac{b^2}{a}$ geeft $x = C_1 e^{0t} + C_2 e^{-\frac{b^2}{a}} t$. PartOpl probeer $x = qt$ invullen geeft $qb^2 = c^3$ dus $q = \frac{c^3}{b^2}$ dus PO: $x = \frac{c^3}{b^2}t$
   e. $\frac{d}{dx}\left\{\cos\left(e^{2x^2}\right)\right\} = \left(-\sin e^{2x^2}\right)\left(e^{2x^2}\right)(2(2x)) = -4xe^{2x^2}\sin e^{2x^2}$

2. kogel
   a. x: $0 = F = ma \Rightarrow \ddot{x} = 0$ en y: $mg = F = ma \Rightarrow \ddot{y} = a$
   b. $x(t) = v_0 t \cos\alpha$ en $y(t) = h + v_0 t \sin\alpha - \frac{1}{2}gt^2$
   c. Dan is $y = 0$ dus als $h + v_0 t \sin\alpha - \frac{1}{2}gt^2 = 0$, de enige fysische oplossing is: $t_g = \frac{v_0}{g}\sin\alpha + \sqrt{\left(\frac{v_0}{g}\sin\alpha\right)^2 + 2\frac{h}{g}}$
   d. $\sin^2\alpha = \frac{2gh}{v_0^2} \Rightarrow \frac{v_0}{g}\sin\alpha = \sqrt{\frac{2h}{g}}$ dus $t_g = \sqrt{\frac{2h}{g}} + \sqrt{\left(\sqrt{\frac{2h}{g}}\right)^2 + 2\frac{h}{g}} = \left(2+\sqrt{2}\right)\sqrt{\frac{h}{g}}$
   e. $x(t) = v_0\left(2+\sqrt{2}\right)\sqrt{\frac{h}{g}}\cos\alpha = v_0\left(2+\sqrt{2}\right)\sqrt{\frac{h}{g}}\sqrt{1-\frac{2gh}{v_0^2}} =$
   $= \left(2+\sqrt{2}\right)\sqrt{\frac{hv_0^2}{g} - 2h^2}$

3. hellend vlak
   a. De verplaatsing van alle massa's is dezelfde dus $a_1 = a_2 = a_3$. BWV: $(m_1+m_2)a = T - (m_1+m_2)g\sin\theta$ en $m_3 a = m_3 g - T$ elimineer T door ze op te tellen:
   $(m_1+m_2+m_3)a = -(m_1+m_2)g\sin\theta + m_3 g$ dus
   $a_{123} = \frac{-(m_1+m_2)g\sin\theta + m_3 g}{m_1+m_2+m_3}$
   b. $F_{w,\max} - m_2 g\sin\theta = m_2 a$. Nu is $F_{w,\max} = \mu_2 N$ en $N = m_2 g\cos\theta$. Dus als $\mu_2 m_2 g\cos\theta - m_2 g\sin\theta = m_2 a = m_2\frac{-(m_1+m_2)g\sin\theta + m_3 g}{m_1+m_2+m_3}$ ofwel
   $\mu_2 = \frac{m_3(1+\sin\theta)}{(m_1+m_2+m_3)\cos\theta}$
   c. NB F_{12} is niet $F_{w,\max}$: hij kan ook kleiner zijn.De versnelling van m_2 is a die is resultaat van zwaartekracht + wrijvingskracht van m_1. Dus BWV $F_{12} - m_2 g\sin\theta = m_2 a$ dus $F_{12} = m_2 a + m_2 g\sin\theta = m_2\frac{-(m_1+m_2)g\sin\theta + m_3 g}{m_1+m_2+m_3} + m_2 g\sin\theta = m_2 m_3 g\frac{1+\sin\theta}{m_1+m_2+m_3}$
   d. De normaal kracht van m_1 op het vlak is $g\cos\theta(m_1+m_2)$.
   De BWV: $(m_1+m_2+m_3)a = -(m_1+m_2)g\sin\theta - \mu_1 g\cos\theta(m_1+m_2) + m_3 g$
   dus $a = \frac{-(m_1+m_2)g\sin\theta - \mu_1 g\cos\theta(m_1+m_2) + m_3 g}{(m_1+m_2+m_3)}$
   e. Zie b.: $\mu_2 m_2 g\cos\theta - m_2 g\sin\theta = m_2 a = m_2\frac{-(m_1+m_2)g\sin\theta - \mu_1 g\cos\theta(m_1+m_2) + m_3 g}{(m_1+m_2+m_3)}$
   dus $\mu_2 = \frac{-\mu_1\cos\theta(m_1+m_2) + m_3(1+\sin\theta)}{(m_1+m_2+m_3)\cos\theta}$

4. Potentiaal
   a , b, c en d



e. $F(x) = -\frac{\partial U}{dx} = -\frac{\partial\left(-x^{-1}\right)}{dx} = -\frac{1}{x^2}$ naar links

5. tunnel
   a. $r = \sqrt{d^2 + x^2}$ en $F_{tot}(r) = \frac{GmM_E}{R_E^3} r$ in de lengterichting van de tunnel is dat $F(x) = F_{tot}(r)\cos\alpha = F_{tot}(r)\frac{x}{r} = \frac{GmM_E}{R_E^3} r \frac{x}{r} = \frac{GmM_E}{R_E^3} x$
   b. $m\ddot{x} = -\frac{GmM_E}{R_E^3} x \Rightarrow \ddot{x} = -\frac{GM_E}{R_E^3} x$. De oplossing is $x = A\cos\omega t$ met $\omega = \sqrt{\frac{GM_E}{R_E^3}}$
   c. $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_E^3}{GM_E}}$ onafhankelijk van d
   d. $W = \int_A^0 \mathbf{F}(x) \cdot d\mathbf{x} = \int_A^0 -\frac{GmM_E}{R_E^3} x dx = \frac{1}{2} A^2 \frac{GmM_E}{R_E^3}$
   e. $P = \mathbf{F} \cdot \mathbf{v} = \frac{-GmM_E}{R_E^3} x \cdot \dot{x} = \frac{GmM_E}{R_E^3} A\cos\omega t \cdot A\omega \sin\omega t = \frac{GmM_E}{2R_E^3} \omega A^2 \sin 2\omega t = \frac{mA^2}{2}\left(\frac{GM_E}{R_E^3}\right)^{\frac{3}{2}} \sin 2\omega t$
   f. $\ddot{x} + \frac{b}{m}\dot{x} + \frac{GM_E}{R_E^3} x = 0$ heeft oplossing $x = Ae^{-\frac{b}{2m}t}\cos\omega t$ amplitude verandert met een factor e^{-1} voor $t = \frac{2m}{b}$
   g. $Q = \frac{\omega_0}{\gamma} = \frac{m}{b}\sqrt{\frac{GM_E}{R_E^3}}$ of $Q = 2\pi \frac{tot\ en}{gediss\_en} = 2\pi \frac{A^2}{A^2 - A^2\left(e^{-\frac{b}{2m}T}\right)^2} = 2\pi \frac{1}{1 - e^{-\frac{2b}{2m}T}} \simeq 2\pi \frac{1}{1 - \left(1 - \frac{b}{m}T\right)} = \frac{2\pi}{bT} m = \frac{2\pi}{b 2\pi \sqrt{\frac{R_E^3}{GM_E}}} m = \frac{m}{b}\sqrt{\frac{GM_E}{R_E^3}}$
   h. Nu is $r = \sqrt{R_E^2 + x^2}$ maar nu is $F_{tot}(r) = \frac{GmM_E}{r^2}$; in de lengterichting van de tunnel is dat $F(x) = F_{tot}(r)\cos\alpha = \frac{GmM_E}{r^2}\frac{x}{r} = GmM_E \frac{x}{r^3} = GmM_E \frac{x}{\left(R_E^2 + x^2\right)^{\frac{3}{2}}}$ en de BWV is $m\ddot{x} = -GmM_E \frac{x}{\left(R_E^2 + x^2\right)^{\frac{3}{2}}}$
   i. Nee want $F(x) \nsim x$ maar $F(x) \sim \frac{x}{\left(C + x^2\right)^{\frac{3}{2}}}$.