Exam Logical Verification

December 18, 2008

There are six (6) exercises.
Answers may be given in Dutch or English. Good luck!

Exercise 1. This exercise is concerned with first-order propositional logic (prop1) and simply typed λ-calculus ($\lambda\!\rightarrow$).

a. Give a proof in prop1 showing that the following formula is a tautology:

$$((B \to A \to B) \to A) \to A$$

(5 points)

b. Give the type-derivation in $\lambda\!\rightarrow$ corresponding to the proof in 1a.
(5 points)

c. Complete the following simply typed λ-terms:

$$\lambda x :?.\, \lambda y :?.\, \lambda z :?.\, x\, z\, y$$
$$\lambda x :?.\, \lambda y :?.\, \lambda z :?.\, x\, (z\, y)$$
$$\lambda x :?.\, \lambda y :?.\, ((\lambda u :?.\, x\, u\, u)\, y)$$

(5 points)

Exercise 2. This exercise is concerned with first-order predicate logic (pred1) and λ-calculus with dependent types (λP).

a. Give a proof in pred1 showing that the following formula is a tautology:

$$\forall x.\, (P(x) \to (\forall y.\, P(y) \to A) \to A)$$

(5 points)

b. Give the λP-term corresponding to the formula in 2a.
(5 points)

c. Give a closed inhabitant in λP of the answer to 2b.
(5 points)

Exercise 3. This exercise is concerned with second-order propositional logic (prop2) and polymorphic λ-calculus ($\lambda 2$).

a. Give a proof in prop2 showing that the following formula is a tautology:

$$(\forall c. (a \to b \to c) \to a) \to a$$

(5 points)

b. Give the $\lambda 2$-type corresponding to the formula of 3a.

(5 points)

c. Give a closed inhabitant in $\lambda 2$ of the answer to 3b.

(5 points)

Exercise 4. This exercises is concerned with encodings.

a. Give an definition of *false* in prop2 and show that the elimination rule for *false* (stating that from *false* follows any proposition) can be derived.

(5 points)

b. We define and $A\,B$ in $\lambda 2$ as follows:

$$\mathsf{and}\,A\,B := \Pi c : *. (A \to B \to c) \to c$$

Assume an inhabitant $P : \mathsf{and}\,A\,B$. Give an inhabitant of A, assuming $A : *$.

(5 points)

c. The datatype of natural numbers is encoded in $\lambda 2$ as

$$\mathsf{Nat} := \Pi a : *. a \to (a \to a) \to a$$

Give two different inhabitants in $\lambda 2$ of this type.

(5 points)

Exercise 5. This definition is concerned with inductive datatypes.

a. Give the definition of an inductive datatype with exactly three elements.

(5 points)

b. Give the definition of an inductive datatype with zero elements.

(5 points)

c. Give the type of the term `natlist_ind`, which gives the induction principle for finite lists of natural numbers.

(5 points)

Exercise 6. This exercise is concerned with inductive predicates.

a. Consider the inductive definition of the predicate `le`:

```
Inductive le (n:nat) : nat -> Prop :=
| le_n : le n n
| le_S : forall m:nat , le n m -> le n (S m) .
```

Give an inhabitant of `le 0 (S 0)`.

(5 points)

b. Consider the inductive definition of the predicate `palindrome`:

```
Inductive palindrome : natlist -> Prop :=
| palindrome_zero :
    palindrome nil
| palindrome_one :
    forall n:nat, palindrome (cons n nil)
| palindrome_more :
  forall n:nat, forall k l : natlist,
  (palindrome l) -> (without_last n k l) -> palindrome (cons n k).
```

How do you write *the list consisting of only* 1 *is a palindrome*?

(5 points)

c. Give an inhabitant of your answer to 6(b).

(5 points)

The final note is (the total amount of points plus 10) divided by 10.