Tentamen Toegepaste Logica

19 april 2002

Answers may be given in Dutch or English. Good luck!

Exercise 1.

a. Show that the formula $(A \to C) \to (B \to C) \to A \to B \to C$ is a tautology of first-order minimal propositional logic. (Give a proof in natural deduction with all assumptions canceled.)

(5 points)

b. Give the (formal) type derivation in simply typed λ-calculus corresponding to the proof of 1a.

(5 points)

c. Give an inhabitant of the type $(A \to C) \to (B \to C) \to A \to B \to C$ different from the term found as anwer to 1b.

(5 points)

d. Give a natural deduction proof of $A \to (A \to A) \to A$ with a detour, and with all assumptions canceled.

(5 points)

Exercise 2.

a. Give a derivation in sequent calculus of
$\vdash ((A \to C) \vee (B \to C)) \to ((A \wedge B) \to C)$

(5 points)

b. For first-order propositional sequent calculus in PVS, we use two commands: `flatten`, corresponding to the rules of the sequent calculus where reading upwards a sequent is transformed into one sequent, and `split`, corresponding to the rules of the sequent calculus where reading upwards a sequent is transformed into two sequents.

Give the proof tree for the PVS derivation of
$\vdash ((A \to C) \vee (B \to C)) \to ((A \wedge B) \to C)$

(You may give a refinement of the PVS proof tree, using more steps where PVS uses only one step.)

(5 points)

c. In PVS specifications can be written using the axiomatic and the definitional approach. Explain the two approaches and explain a drawback of each of them.

   (5 points)

d. In PVS all functions are total. What is a way out if we want to represent a partial function? Give a drawback of this approach.

   (5 points)

Exercise 3.

a. A feature of Coq is program extraction. Explain briefly the general principle of program extraction.

   (5 points)

b. Give an example of a formula that is a tautology in classical logic but not in intuitionistic logic.

   (5 points)

Exercise 4.

a. Give the two kinds of detours in first-order predicate logic.

   (5 points)

b. Show that the formula $((\exists x. P(x)) \to B) \to \forall x. (P(x) \to B)$ is a tautology of first-order predicate logic. (Give a derivation in (intuitionistic) natural deduction with all assumptions canceled.)

   (5 points)

Exercise 5.

a. Give the definition of the inductive type `natlist` of finite lists of natural numbers. (The type of natural numbers is `nat`.)

   (5 points)

b. What is the type of a predicate on `natlist`? (A predicate is a propositional function.)

   (5 points)

c. Let `polylist` be the inductive type of polymorphic lists with constructors `polynil` and `polycons`.

   What is the type of `polynil`? And what is the type of `polycons`?

   (5 points)

Exercise 6. This exercise is concerned with $\lambda 2$, the polymorphic λ-calculus.

a. Give the Curry-Howard-de Bruijn correspondence between formulas in second-order propositional logic on the one hand and types of the polymorphic λ-calculus on the other hand.

   (5 points)

b. Suppose that we have $M : \Pi a : \texttt{Set}.a$. Show that for any type $A : \texttt{Set}$ there is a term of type A.

   (5 points)

c. The function $\mathsf{Pair} = \lambda n{:}\texttt{nat}.\,\lambda \texttt{m}{:}\texttt{nat}.\,\lambda \texttt{z}{:}\texttt{nat} \to \texttt{nat} \to \texttt{nat}.\,\texttt{z}\,\texttt{n}\,\texttt{m}$ is the pairing function on natural numbers: given two inputs which are natural numbers it forms a pair of them.

   Give the polymorphic version of this function, where the two elements of the pair have the same type.

   (5 points)

The final note is (the total amount of points plus 10) divided by 10.

Appendix: rules of first-order propositional sequent calculus

$$\Gamma, A \vdash A, \Delta$$

$$\frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ R\to$$

$$\frac{B, \Gamma \vdash \Delta \qquad \Gamma \vdash A, \Delta}{A \to B, \Gamma \vdash \Delta}\ L\to$$

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ R\wedge$$

$$\frac{A, B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta}\ L\wedge$$

$$\frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ R\vee$$

$$\frac{A, \Gamma \vdash \Delta \qquad B, \Gamma \vdash \Delta}{A \vee B, \Gamma \vdash \Delta}\ L\vee$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ R\neg$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ L\neg$$

$$\frac{\Gamma_1 \vdash \Delta_1}{\Gamma_2 \vdash \Delta_2}\ w$$

(with $\Gamma_1 \subseteq \Gamma_2$ and $\Delta_1 \subseteq \Delta_2$)

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma \vdash A, \Delta}{\Gamma \vdash \Delta}\ c$$