Opgave 1

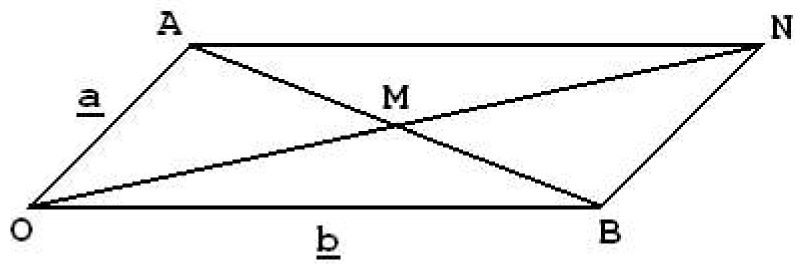
(a) $$a - b = \begin{pmatrix} 3 \\ 7 \\ 9 \end{pmatrix} - \begin{pmatrix} 17 \\ 9 \\ 3 \end{pmatrix} = \begin{pmatrix} -14 \\ -2 \\ 6 \end{pmatrix} = 14i - 2j + 6k$$

(b) $$|a - b| = \sqrt{\left(14^2 + 2^2 + 6^2\right)} = \sqrt{236}$$

(c)
$$\cos\alpha = \frac{14}{\sqrt{236}}$$
$$\cos\beta = \frac{-2}{\sqrt{236}}$$
$$\cos\gamma = \frac{6}{\sqrt{236}}$$

$$\alpha = \cos^{-1}\left(\frac{-14}{\sqrt{236}}\right)$$
$$\beta = \cos^{-1}\left(\frac{-2}{\sqrt{236}}\right)$$
$$\chi = \cos^{-1}\left(\frac{-2}{\sqrt{236}}\right)$$

Opgave 2



OA = **a**
OB = **b**
M is snijpunt AB en ON

AB = -**a** + **b** = **b** - **a**
ON = **b** + **a**

Te Bewijzen:
AM = ½AB = ½(**b** - **a**) en OM = ½ON = ½(**b** + **a**)

Bewijs:

$OM = \lambda ON = \lambda(\mathbf{b} + \mathbf{a})$

$AM = \mu AB = \mu(\mathbf{b} - \mathbf{a})$

$OM - AM = \mathbf{a}$

$\lambda(\mathbf{b} + \mathbf{a}) - \mu(\mathbf{b} - \mathbf{a}) = \mathbf{a}$

$\lambda\mathbf{b} + \lambda\mathbf{a} - \mu\mathbf{b} + \lambda\mathbf{a} = \mathbf{a}$

$\underbrace{\lambda\mathbf{b} - \lambda\mathbf{b}}_{\lambda=\mu} + \underbrace{\mathbf{a} + \lambda\mathbf{a} = \mathbf{a}}_{\lambda+\mu=1}$

$\lambda = \mu = \frac{1}{2}$

Dus OM = ½ON en AM = ½AB

Opgave 3

$a = a_1 i + a_2 j + a_3 k$

$b = b_1 i + b_2 j + b_3 k$

$(a_1 i + a_2 j + a_3 k) \times (b_1 i + b_2 j + b_3 k)$

$= a_1 b_2 ij + a_1 b_3 ik + a_2 b_1 ji + a_2 b_3 jk + a_3 b_1 ki + a_3 b_2 kj$

$= a_1 b_2 k + a_1 b_3 j - a_2 b_1 k + a_2 b_3 i + a_3 b_1 j - a_3 b_2 i$

$= (a_2 b_3 i - a_3 b_2 i) + (a_3 b_i j - a_1 b_3 j) + (a_1 b_2 k - a_2 b_1 k)$

Opgave 4

Afstand tussen vlak 1x + 2y + 3z en oorsprong $\begin{pmatrix}0\\0\\0\end{pmatrix}$

"kortste afstand staat loodrecht op het vlak in de richting van de normaal"

Normaal n = $\begin{pmatrix}1\\2\\3\end{pmatrix}$

$Q = P + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = 5$ (gegeven)

$$\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = 5$$

$$\lambda(1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3) = 5$$

$$14\lambda = 5$$

$$\lambda = \frac{5}{14}$$

$$PQ = Q - P = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} = \frac{5}{14}\begin{pmatrix}1\\2\\3\end{pmatrix}$$

$$|PQ| = \frac{5}{14} \cdot \sqrt{(1^2 + 2^2 + 3^2)} = \frac{5}{14} \cdot \sqrt{14}$$

Opgave 5

$$\left|A^T\right| = |A| = \begin{bmatrix}1 & 2\\1 & 4\end{bmatrix} = (1 \cdot 4) - (2 \cdot 1) = 2$$

$$\left|A^7\right| = |A|^7 = 2^7 = 128$$

Opgave 6

?

Opgave 7

$$x_1 + x_2 + x_3 = 7$$
$$x_1 + 2x_2 - x_3 = p$$
$$2x_1 + qx_2 + 4x_3 = 16$$

$$\left(\begin{array}{ccc|c}1 & 1 & 1 & 7\\1 & 2 & -1 & p\\2 & q & 4 & 16\end{array}\right) \rightarrow (R_2 - R_1) \rightarrow \left(\begin{array}{ccc|c}1 & 1 & 1 & 7\\0 & 1 & -2 & p-7\\2 & q & 4 & 16\end{array}\right) \rightarrow (R_3 - 2R_1) \rightarrow$$

$$\left(\begin{array}{ccc|c}1 & 1 & 1 & 7\\0 & 1 & -2 & p-7\\0 & q-2 & 2 & 2\end{array}\right) \rightarrow (R_3 + R_2) \rightarrow \left(\begin{array}{ccc|c}1 & 1 & 1 & 7\\0 & 1 & -2 & p-7\\0 & q-1 & 0 & p-5\end{array}\right)$$

- Geen oplossing als (q – 1 = 0) en (p – 5 != 0)
- Oneindig aantal oplossing als (q – 1 = 0) en (p – 5 = 0)
- Eén oplossing als (q – 1 != 0)