UITWERKING VOORTENTAMEN WISKUNDE I 20-10-1998

1 a) $(z-\sqrt{3})^2 = 2+2i\sqrt{3} = 4\left(\frac{1}{2}+\frac{1}{2}i\sqrt{3}\right) = 4e^{(\pi/3)i}$

dus $z-\sqrt{3} = \pm 2e^{(\pi/6)i} = \pm 2\left(\frac{1}{2}\sqrt{3}+\frac{1}{2}i\right) = \pm(\sqrt{3}+i)$

dus $z_1 = 2\sqrt{3}+i$ en $z_2 = -i$

b) $z^4 = -9 = -9e^{\pi i}$. 1) $|z^4| = |z|^4 = |-9| = 9$ dus $|z| = \sqrt{3}$

2) $\arg z^4 = 4\arg z = \arg(-9) = \pi$ dus $\arg z = \frac{\pi}{4}$.

dus $z_1 = \sqrt{3}e^{(\pi/4)i} = \frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{6}\,i$ $\quad z_2 = \sqrt{3}e^{(3\pi/4)i} = -\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{6}\,i$

$z_3 = \sqrt{3}e^{(-\pi/4)i} = \frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{6}\,i$ $\quad z_4 = \sqrt{3}e^{(-3\pi/4)i} = -\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{6}\,i$

c) Stel $z = x+i\cdot y$. Dan: ① $|x+iy+i-1| = |x+iy-i+1|$

dus $(x-1)^2+(y+1)^2 = (x+1)^2+(y-1)^2$

dus $\cancel{x^2}-2x+\cancel{1}+\cancel{y^2}+2y+\cancel{1} = \cancel{x^2}+2x+\cancel{1}+\cancel{y^2}-2y+\cancel{1}$

dus $\boxed{x=y}$

② (Gebruik resultaat ①, dus $z = x+i\cdot x$).

Dan: $|x+3+i(x-1)| = 4$, dus $(x+3)^2+(x-1)^2 = 16$

dus $x^2+2x-3=0$, dus $x=1$ of $x=-$

Dus ①+② levert: $z = 1+i$ of $z = -3-3i$

d) (i) $|z| = \sqrt{4+4} = 2\sqrt{2}$ en $\arg z = \pi/4$

(ii) 1) $|z|^{11} = (2\sqrt{2})^{11} = 2^{16}\sqrt{2}$

2) $\arg z^{11} = 11\cdot\arg z = \frac{11}{4}\pi \equiv \frac{3}{4}\pi$

$\Big\}\; z^{11} = 2^{16}\sqrt{2}\,e^{\frac{3}{4}\pi i} = 2^{16}(-1+i$

2 a) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & a & 3 \\ 4 & a+2 & a \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & a-2 & 1 \\ 4 & a-2 & a-4 \end{vmatrix} = \begin{vmatrix} a-2 & 1 \\ a-2 & a-4 \end{vmatrix} = (a-2)(a-$

dus $a=2$ of a

b) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 5 & 3 & 5 \\ 4 & 7 & 5 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 3 & 1 & 5 \\ 0 & 3 & 1 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 3 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$

Kies x_3 vrij. Dan $3x_2+x_3=5$, dus $x_2 = -\frac{1}{3}x_3+\frac{5}{3}$

$x_1+x_2+x_3=0$, dus $x_1 = -\frac{2}{3}x_3-\frac{5}{3}$

$\Big\}\; \underline{x} = x_3\cdot\begin{pmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix}$

c) $\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 6 & 3 & 0 & 1 & 0 \\ 4 & 8 & 6 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & -2 & 1 & 0 \\ 0 & 4 & 2 & -4 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right.$

$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 3 & 1 & -1 \\ 0 & 4 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1/2 & -3/4 \\ 0 & 1 & 0 & 0 & 1/2 & -1/4 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{array}\right)$ dus $A^{-1} = \left(\begin{array}{ccc} 3 & 1/2 & -3/4 \\ 0 & 1/2 & -1/4 \\ -2 & -1 & 1 \end{array}\right.$

3 a) $\left(\begin{array}{ccc|c} -2 & 1 & 1 & 8 \\ -1 & 2 & 1 & 7 \\ 0 & 3 & 1 & h \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & 1 & 8 \\ 0 & \frac{3}{2} & \frac{1}{2} & 3 \\ 0 & 3 & 1 & h \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & 1 & 8 \\ 0 & \frac{3}{2} & \frac{1}{2} & 3 \\ 0 & 0 & 0 & h-6 \end{array}\right)$

Dus als $h \neq 6$ is er een strijdig stelsel, dus geen oplossingen.
Als $h = 6$ zijn er (∞ veel) oplossingen.

b) Na vegen blijkt er een rij met nullen te verschijnen, dus det B =
Maar det B = det$(B - 0 \cdot I) = 0$. Dus $\lambda = 0$ is een oplossing van det$(B - \lambda I) = 0$. Dus λ is een eigenwaarde van B.

c) $\left|\begin{array}{ccc} -2-\lambda & 1 & 1 \\ -1 & 2-\lambda & 1 \\ 0 & 3 & 1-\lambda \end{array}\right| = -3 \cdot \left|\begin{array}{cc} -2-\lambda & 1 \\ -1 & 1 \end{array}\right| + (1-\lambda)\left|\begin{array}{cc} -2-\lambda & 1 \\ -1 & 2-\lambda \end{array}\right| =$

$= -3(-2-\lambda+1) + (1-\lambda)(\lambda^2 - 4 + 1) = -\lambda^3 + \lambda^2 + 6\lambda = 0$

dus $\lambda_1 = 0$ of $\lambda^2 - \lambda - 6 = 0$, dus $\lambda_2 = 3$ en $\lambda_3 = -2$

bij $\lambda_1 = 0$: $\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, dus bijv $\underline{x}_1 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

bij $\lambda_2 = 3$: $\left(\begin{array}{ccc|c} -5 & 1 & 1 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 3 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -5 & 1 & 1 & 0 \\ 0 & -4/5 & 4/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, dus bijv $\underline{x}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

bij $\lambda_3 = -2$: $\left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ -1 & 4 & 1 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ -1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, dus bijv $\underline{x}_3 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

d) $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ en $S = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 3 & -1 \end{pmatrix}$.