

Final Exam: Probability and Statistics (code 400178)

Vrije Universiteit Amsterdam

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You are allowed to use a calculator, but not the book or your notes. You can express your numerical answers as fractions.

Each question is worth 2 points (if answered completely and correctly). The total number of points is 40 (corresponding to 20 questions, divided in 8 problems). Give clear answers to as many questions as you can.

Problem 1. You and 9 other people are at small but fancy cocktail party where they serve 100 different cocktails. Suppose that each person is drinking a cocktail and that the different cocktails are all equally likely to be chosen.

- a) What is the probability that two persons are drinking the same cocktail?
- b) What is the probability that someone else is drinking the same cocktail as you?

Problem 2. In a game of chance, your probability of winning is p . Suppose that you play three times, and consider the following events:

$A = \{\text{you win the first game}\}$, $B = \{\text{you win the second game}\}$ and $C = \{\text{you win exactly once}\}$.

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- a) For what value of p is C independent of A and B ? Motivate your answer.
 - b) For $0 < p < 1$, can the three events ever be mutually independent (i.e., $P(A, B, C) = P(A)P(B)P(C)$)? Motivate your answer.

Problem 3. A die is thrown six times and the face values that come up are recorded. Let $A = \{\text{the first value is 6}\}$, $B = \{\text{the second value is 6}\}$ and $C = \{6 \text{ comes up exactly once}\}$.

- a) Indicate all pairs of events that are independent? Motivate your answer.
- b) Are the three events mutually independent? Motivate your answer.

Problem 4. A fair coin is flipped n times. You can give your answers as sums.

- a) What is the probability to get exactly k heads?
- b) What is the probability to get at least k heads?
- c) Suppose that n is odd. What is the probability to get one more head than tails?

Problem 5. Let X be an exponential random variable with density function $f(x) = \lambda e^{-\lambda x}$ if $x \geq 0$ and $f(x) = 0$ if $x < 0$.

- a) Write the cumulative distribution function (cdf) $F(x)$.
- b) For what number y is $P(X < y) = 1/2$?

Problem 6. A box contains N identical balls. You extract k balls, mark them, and put them back inside the box. After shaking the box you extract $k + 1$ balls randomly, k of which are marked.

- a) What is the probability of the event described?
- b) Suppose that $k = 1$. What is your “maximum likelihood” estimate for N ? Motivate your answer.

Problem 7. Suppose that the temperatures measured in Celsius at a particular time of the year and geographic location are distributed according to a Normal distribution with $\mu = 30$ and $\sigma = 2$. An American tourist planning to visit the place translates the temperatures into Fahrenheit by multiplying by $9/5$ and then adding 32.

- a) According to what type of distribution are the Fahrenheit temperatures distributed?

- b) What are the expected value and variance of the temperatures in Fahrenheit?

Problem 8. Eva works at a supermarket where the arrival of customers happens according to a Poisson process and the expected number of customers is one per minute ($\lambda = 1$). Suppose that at 9:00 Eva serves her first customer.

- a) What is the probability that no other customer arrives in the next minute?
- b) What is the expected value of the time T between two successive arrivals?
- c) What is the probability that more than three customers arrive between 9:30 and 9:31?

It is 10:32 and Eva's last customer arrived at 10:30.

- d) What is the probability that no customer arrives until 10:35?
- e) It is now 10:35 and no customer has arrived. What is the probability that the next customer arrives before 10:37?