Remarks:
The book and other study material may be used during the exam.
It is allowed to use calculators.
You may submit your answers in English or Dutch.
Good luck!

1. From a shuffled deck of 52 cards, two cards are drawn at random.

   (a) Give an appropriate sample space and probability measure for this experiment.

   (b) What is the probability that the first card is an ace, and the second one is hearts?

   (c) Are the events "first card is an ace" and "second card is hearts" independent? Motivate your answer.

2. Let X be a continuous random variable with density f given by

$$f(x) = \begin{cases} Cx^4, & x \in [1,2], \\ 0, & \text{elsewhere}, \end{cases}$$

   where $C > 0$ is a constant to be determined later.

   (a) Determine the value of the constant C for which f is a density.

   (b) Compute the expectation $\mathbb{E}X$, the second moment $\mathbb{E}X^2$ and the variance $\text{Var}X$ of X.

3. Let (X, Y) be a jointly continuous random vector with joint density f given by

$$f(x,y) = \begin{cases} 4xy - 8y, & x \in (2,3), \ y \in (0,1), \\ 0, & \text{elsewhere}. \end{cases}$$

   (a) Determine the marginal densities of X and Y.

   (b) Are X en Y independent? Motivate your answer.

4. Let $X_1, \ldots, X_n$ be independent, continuous random variables, each with density

$$f(x \mid \theta) = \begin{cases} (1+\theta)x^\theta, & x \in (0,1), \\ 0, & \text{elsewhere}, \end{cases}$$

where $\theta > -1$ is an unknown parameter.

(a) Find the method of moments estimator for θ.

(b) Find the maximum likelihood estimator for θ.

5. Let X be a binomial random variable with parameters $n = 20$ and $p \in (0,1)$ (p is unknown). Give a test with significance level $\alpha = 5\%$ for the hypotheses

$$H_0 : p = 0.4,$$

$$H_A : p > 0.4.$$

Maximal number of points:

1(a):	2	2(a):	3	3(a):	4	4(a):	3	5: 5
1(b):	4	2(b):	4	3(b):	3	4(b):	4	
1(c):	4							

Grade = (points+4)/4