Afdeling Wiskunde	Tentamen Kansrekening en Statistiek
Vrije Universiteit	1 juni 2004

Remarks:
The book and other study material may be used during the exam.
It is allowed to use calculators.
You may submit your answers in English or Dutch.
Good luck!

1. Consider a pond which contains an unknown number, say N, of fishes. Suppose that 50 of them have been tagged. We do the following experiment: we catch 100 fishes at random and see if they are tagged.

   (a) Give an appropriate sample space and probability measure for this experiment.

   (b) Let X denote the number of tagged fishes we have caught. Determine the probability mass function/frequency function of the random variable X.

2. Let X be a continuous random variable with density f given by

$$f(x) = \begin{cases} \dfrac{C}{x^2}, & x \geq 1, \\ 0, & \text{elsewhere}, \end{cases}$$

   where $C > 0$ is a constant to be determined later.

   (a) Determine the value of the constant C for which f is a density.

   (b) Compute the expectation $\mathbb{E}X$, the second moment $\mathbb{E}X^2$ and the variance $\mathbb{V}\text{ar}X$ of X.

3. Let (X, Y) be a jointly continuous random vector with joint density f given by

$$f(x, y) = \begin{cases} 4xy - 4x, & x \in (0, 1), \ y \in (1, 2), \\ 0, & \text{elsewhere}. \end{cases}$$

   (a) Determine the marginal densities of X and Y.

   (b) Are X en Y independent? Motivate your answer.

4. Let $X_1, \ldots, X_n$ be independent, continuous random variables, each with density

$$f(x \mid \theta) = \begin{cases} \theta \dfrac{1}{x^{1+\theta}}, & x > 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\theta > 1$ is an unknown parameter.

(a) Find the method of moments estimator for θ.

(b) Find the maximum likelihood estimator for θ.

5. Let X be a binomial random variable with parameters $n = 25$ and $p \in (0,1)$ (p is unknown). Give a test with significance level $\alpha = 5\%$ for the hypotheses

$$H_0 : p = 0.6,$$

$$H_A : p < 0.6.$$

Maximal number of points:

1(a): 4 2(a): 3 3(a): 4 4(a): 3 5: 6
1(b): 4 2(b): 4 3(b): 3 4(b): 5

Grade = (points+4)/4