Dept. Computer Science	**Networks and Graphs**
VU University Amsterdam	**27.05.2014**

BE SURE THAT YOUR HANDWRITING IS READABLE

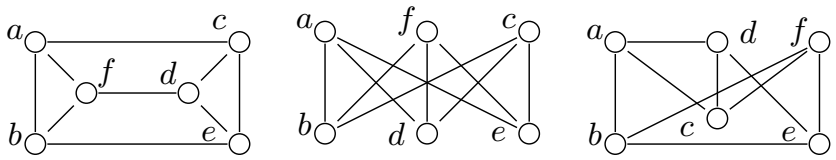
Part I

1a Let G denote a simple graph with n vertices and m edges. For each of the following mathematical statements, (1) translate the statement into common English and (2) tell whether it is true or false.

1. $\forall a, b \in V(G) : \langle a, b\rangle \in E(G)$ or $\exists c : \langle a, c\rangle, \langle c, b\rangle \in E(G)$.
2. $\forall k, l \in \mathbb{N} : \chi(K_{k,l}) = \min(k, l)$.
3. $\forall H \subseteq G : \sum_{v \in V(H)} \delta(v) \leq 2 \cdot m$.
4. $\forall H \subseteq G : |E(H)| \leq |E(G[V(H)])|$.
5. G is Hamiltonian $\Rightarrow \forall S \subset V(G)$ and $S \neq \emptyset$, $\omega(G - S) \leq |S|$.

10pt

2a Determine which of the following graphs are isomorphic. Be sure to explain your answer; if graphs are isomorphic, provide ϕ.



G_1 G_2 G_3

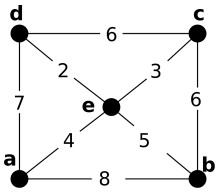
7pt

2b Provide an algorithm for checking whether an undirected graph G is connected. *6pt*

3a Prove that if each component of a graph is bipartite, then the entire graph is bipartite. *5pt*

3b Prove that a bipartite graph with an odd number of vertices cannot contain a Hamilton cycle. *7pt*

4a Find a closed walk of minimal weight in the following graph by applying the Chinese postman algorithm. Be sure to explains the steps (according to the algorithm) in your answer. *6pt*



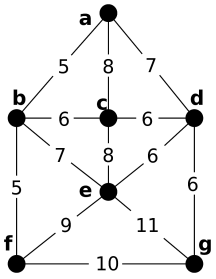
4b Consider the complete bipartite graph $K_{m,n}$, with $m, n > 0$. For which values of m and n, is $K_{m,n}$ Eulerian? *3pt*

5 Prove that every planar graph with $n \geq 3$ vertices and m edges has a vertex of degree at most 5. *6pt*

Part II

6a Prove that a connected graph G is a tree if and only if every edge is a cut edge. *7pt*

6b Find the weight of a minimum spanning tree in the following graph, using Kruskal's algorithm. Be sure to explains the steps (according to the algorithm) in your answer.



5pt

7a Explain that the probability $\mathbb{P}[\delta(u) = k]$ in *ER*(n, p) graph is

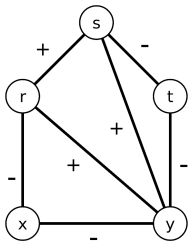
$$\binom{n-1}{k} p^k (1-p)^{n-1-k}.$$

7pt

7b Compute the expected network density $\rho(G)$ of $G = ER(50, 0.4)$. Explain your answer. *7pt*

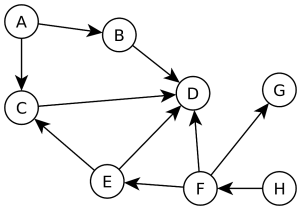
8 Prove by induction that the number of triples at a vertex $n_\Lambda(v) = \binom{\delta(v)}{2}$. *10pt*

9a Use the balanced graph algorithm to check if the following graph is balanced:



Be sure to include all of the iterations of the algorithm in your answer. *7pt*

9b Compute proximity prestige for vertices D, E and H of the following graph. Explain your answer.



7pt

Final grade: *(1) Add, per part, the total points. (2) Let T denote the total points for the midterm exam ($0 \le T \le 50$); $D1$ the total points for part I; $D2$ the total points for part II. The final number of points E is equal to* $\max\{T, D1\} + D2$.