Afdeling Informatica
Vrije Universiteit

Exam Introduction to Logic
December 17, 2007

This exam has 3 pages; there are 8 exercises.

For each item the number of points that can be obtained is indicated. The final result is: (total number of points plus 10) divided by 10.

Exercise 1. Give derivations with natural deduction (ND):

(a) $(p \wedge q) \vee \neg\neg q \;\vdash\; q$ (5 points)

(b) $(q \vee p) \rightarrow r \;\vdash\; \neg r \rightarrow \neg p$ (5 points)

Exercise 2.

(a) Find a conjunctive normal form (CNF) for the formula

$$(\neg p \rightarrow q) \vee \neg(p \vee r)$$

Show how you found the answer. (6 points)

(b) Give 3 formulas of propositional logic $\varphi_1, \varphi_2, \varphi_3$, such that $\varphi_1 \models \varphi_2$ and $\varphi_2 \models \varphi_3$ hold, but on the other hand $\varphi_3 \not\models \varphi_2$ and $\varphi_2 \not\models \varphi_1$.

(4 points)

Exercise 3. Given is the boolean function $f(x, y)$ that is 1 if both variables x, y have the value 0, and 0 otherwise.
Also given is the boolean function $g(z, w)$ that is 1 if at least one of the variables z, w have the value 1, and 0 otherwise.

(a) Give a reduced BDD for the boolean function f w.r.t. the variable ordering $[x, y]$; and a reduced OBDD of the function g w.r.t. the ordering $[z, w]$. (8 points)

(b) Now give a reduced OBDD of the boolean function $\overline{f(x, y)} \cdot \overline{g(z, w)}$, with variable ordering $[x, y, z, w]$.

Hint: use (a) (5 points)

Exercise 4. Give derivations with natural deduction (ND):

(a) $\forall x\,(Px \rightarrow Qx), \forall x Px \vdash \forall x Qx$ (5 points)

(b) $\forall x(Px \rightarrow \neg\exists y Rxy) \vdash \neg\exists x\,(Px \wedge Rxx)$ (5 points)

Exercise 5. The following entailments are not valid. Show this by giving countermodels.

(a) $\forall x \forall y(Rxy \vee Ryx) \vDash \exists x \forall y Rxy$ (5 points)

(b) $\forall x \exists y Rxy, \forall x \forall y\,(Rxy \rightarrow \neg Ryx) \vDash$
$\exists x \exists y \exists z \exists w\,(x \neq y \wedge x \neq z \wedge x \neq w \wedge y \neq z \wedge y \neq w \wedge z \neq w)$
(5 points)

Exercise 6. Translate the following sentences into the the language of predicate logic. Make use of the key:

Bx:	x is a book
Kxy:	x has bought y
Lxy:	x has read y
a:	Anna
c:	Chris

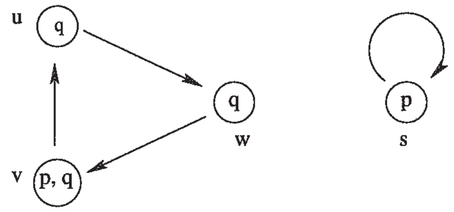
(a) Chris has bought several books that he hasn't read.
(Hint: several = at least two.) (4 points)

(b) Anna and Chris have read the same books. (4 points)

(c) Anna and Chris have read the same books but one. (4 points)

Exercise 7.

Given is the Kripke model $\mathcal{M} = (W, R, L)$ with underlying frame $\mathcal{F} = (W, R)$, depicted as follows.



(a) Determine for which worlds x holds:

(i) $\mathcal{M}, x \Vdash p \to \Box p$

(ii) $\mathcal{M}, x \Vdash \Box p \to \Diamond q$

Motivation is not required. (6 points)

(b) Give if possible a labeling function L' on the frame $\mathcal{F}$, such that for the Kripke model $\mathcal{M}' = (W, R, L')$ holds: $\mathcal{M}' \not\Vdash \Diamond q \to q$. If no such L' exists, argue why not. (5 points)

(c) Give if possible a labeling function L'' on the frame $\mathcal{F}$, such that for the Kripke model $\mathcal{M}'' = (W, R, L'')$ holds: $\mathcal{M}'' \not\Vdash \Diamond p \to \Box p$. If no such L'' exists, argue why not. (5 points)

Exercise 8.

(a) Formulate the Correctness Theorem for predicate logic. (2 points)

(b) Use the Correctness Theorem to prove that from the assumption that the set of formulas formules $\{\varphi_1, \dots, \varphi_n\}$ is consistent (i.e. that there is a model $\mathcal{M}$ is such that $\mathcal{M} \models \{\varphi_1, \dots, \varphi_n\}$) follows that $\varphi_1, \dots, \varphi_n \nvdash \bot$. (4 points)

(c) Explain what it means that Predicate Logic is undecidable. (3 points)