
This exam has 3 pages; there are 8 exercises.

For each item the number of points that can be obtained is indicated. The final result is: (total number of points plus 10) divided by 10.

Opgave 1. Give derivations with natural deduction (ND):

(a) $\neg(p \vee q), r \vdash \neg(r \to p)$, (5 points)

(b) $(p \to r) \vee \neg q \vdash (p \wedge q) \to r$. (5 points)

Opgave 2.

(a) Give the criterion (from the book) for determining whether a conjunctive normal form (CNV) is a tautology. (2 points)

(b) Find a CNV for the formula $(\neg p \to \neg q) \vee (\neg(p \wedge \neg r) \wedge (s \to q))$. Show the way you found it. (4 points)

(c) Apply the criterion from (a) to determine whether the formula in (b) is a tautology

(4 points)

Opgave 3. Consider the boolean function $f(x, y)$ which has value 1 if exactly one of the variables x, y is 1, and value 0 in all other cases.
Consider also the boolean function $g(z, w)$ which has value 1 if exactly one of the variables z, w is 1, and value 0 in all other cases.

(a) Find a reduced OBDD for the function f w.r.t. de variable ordering $[x, y]$; and a reduced OBDD for the function g w.r.t. the ordering $[z, w]$.

(5 points)

(b) Find a reduced OBDD for the boolean function $f(x, y) \cdot g(z, w)$, with variable ordering $[x, y, z, w]$. (Hint: use (a))

(5 points)

(c) Consider the boolean function $h(x, y, z, w)$ of four variables x, y, z, w, which is 1 if exactly two of the variables x, y, z, w have value 1, and 0 otherwise. Give a reduced OBDD for h with variable ordering $[x, y, z, w]$.

(5 points)

Opgave 4. Prove or refute the following statements (for propositional logic).

(a) If $\phi \models \psi$ and ϕ is a tautology (i.e. a valid formula), then also ψ is a tautology.
(5 points)

(b) If $\phi \vee \psi$ is a tautology and ϕ is not a tautology, then ψ is a tautology.
(5 points)

Opgave 5. Give derivations with natural deduction (ND):

(a) $\vdash \forall x(Px \vee \neg Px)$, (5 points)

(b) $\forall x(Px \to Sx), \exists x(Px \wedge \neg Qx) \vdash \neg \forall x(Sx \to Qx)$. (5 points)

Opgave 6. The following entailment is not valid. Show this by giving a countermodel.

$$\exists x \forall y Rxy, \forall x Rxx \models \forall x \forall y (Rxy \vee Ryx).$$

(8 points)

Opgave 7. Translate the following sentences into the the language of predicate logic. Make use of the key:

Hx: x is at home
Wx: x works
Sxy: x is a sister of y
j: John

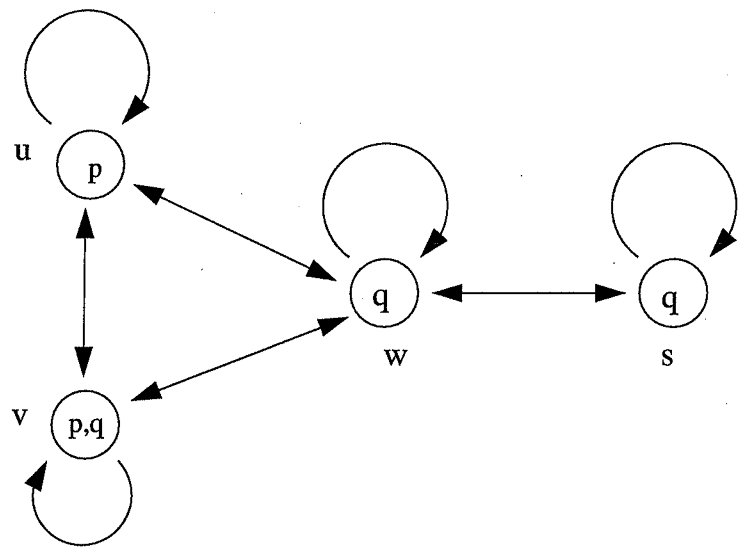
(a) If all his sisters are at home, John does not work.
(4 points)

(b) Several of John's sisters work at home.
(4 points)

(c) Exactly one of John's sisters works at home.
(4 points)

Opgave 8.

Given is the Kripke model $\mathcal{M} = (W, R, L)$ with underlying frame $\mathcal{F} = (W, R)$, depicted as follows.



(a) Determine for which worlds x holds: $\mathcal{M}, x \Vdash \Box q$.
(3 points)

(b) Determine for which worlds x holds: $\mathcal{M}, x \Vdash \Box(\Box q \rightarrow p)$.
(4 points)

(c) Is there a labeling function L' on the frame $\mathcal{F}$, such that for the Kripke-model $\mathcal{M}' = (W, R, L')$ we have: $\mathcal{M}' \nVdash p \rightarrow \Box\Diamond p$? Motivate your answer.
(4 points)

(d) Is there a labeling function L'' on the frame $\mathcal{F}$, such that for the Kripke-model $\mathcal{M}'' = (W, R, L'')$ we have: $\mathcal{M}'' \nVdash \Box p \rightarrow \Box\Box p$? Motivate your answer.
(4 points)