Faculty of Sciences	Discrete Mathematics
VU University Amsterdam	**Mopping up exam 18:30-21:15 5-2-2015**

(1) This exam consists of 12 problems.
(2) For problems 7b and 9 you may use the separate worksheet. **If you do this, write your name on it and hand it in.**
(3) Books, notes, calculators, etc., may not be used.
(4) Give explanations about and reasoning for your solutions.

Problem 1

(a) How many different strings of letters can you form by rearranging the letters in the word "VEEGTENTAMEN"? Give this number as an expression involving factorials and justify it, but do not evaluate it to an explicit number.

(b) (i) How many 3-tuples (x_1, x_2, x_3) of non-negative integers are there that satisfy $x_1 + x_2 + x_3 = 9$?

(ii) For how many of those are x_1, x_2 and x_3 all distinct?

Problem 2

Each working day, a small factory makes 2, 3 or 5 chairs. Assume that a year consists of 52 weeks, and that each week has five working days. Show that a year will contain at least four weeks for which the total number of chairs made during each of those weeks is the same. (Hint: first work out how many chairs can be made in total in five working days.)

Problem 3

A company has to provide office space to six different employees (1,. . . ,6). However, the following pairs of employees cannot get along so must not share an office: $\{1, 5\}$, $\{2, 3\}$, $\{2, 4\}$, $\{2, 5\}$, $\{2, 6\}$, $\{3, 4\}$, $\{3, 6\}$.

(a) What is the minimum number of offices required?

(b) If at most two employees can share an office, can the problem be solved using this minimum number of offices?

Problem 4

(a) Solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2} + 2$ $(n \geq 2)$ with $a_0 = 6$ and $a_1 = 2$.

(b) Use your answer to compute a_{40}.

Problem 5

The generating function $G(x)$ satisfies $(x + 2)G(x) = x^2G(x) + 2$. Express the coefficient of x^k $(k \geq 0)$ in $G(x)$ in terms of k.

Problem 6

Determine how many of the integers 1,. . . ,10000 are divisible by none of 10, 11 and 12.

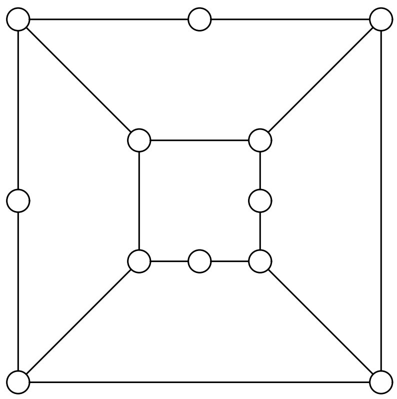
Problem 7

(a) We remove five edges from the complete graph on six points, K_6, obtaining a graph G.

  (i) What is the maximum number of connected components that G can have?

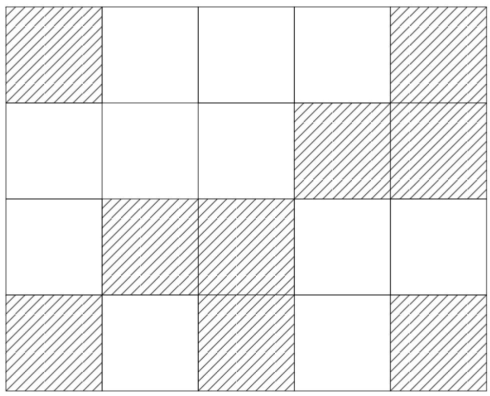
  (ii) Show that G contains a circuit of length 3.

(b) Does the following graph have a Hamiltonian cycle? Either give such a cycle, or give an argument why it does not exist.



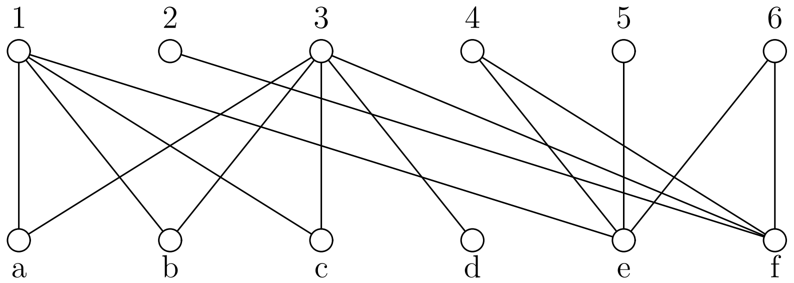
Problem 8

Use the reduction method discussed in class in order to compute the rook polynomial of the following board. (The board consists of the shaded squares.)



Problem 9

Let $G = (X, Y, E)$ with $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{a, b, c, d, e, f\}$ be the bipartite graph below. Starting with the matching M consisting of $\{1, c\}$ and $\{6, f\}$, use the algorithm discussed in class to find a maximum matching for G. What is $\delta(G)$? Also, find an explicit subset S of X that realizes $\delta(G)$, as well as a minimum covering for G.



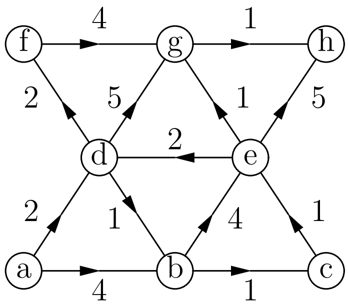
Problem 10

The following matrix gives the cost r_{ij} of worker i on job j. Use the Hungarian algorithm to find an optimum (i.e., with lowest total cost) job assignment.

$$\begin{pmatrix} 4 & 2 & 5 & 5 \\ 2 & 5 & 2 & 3 \\ 3 & 1 & 7 & 5 \\ 2 & 3 & 5 & 4 \end{pmatrix}$$

Problem 11

Use Dijkstra's algorithm to find a shortest path from vertex a to vertex h in the following directed network.



Problem 12

Consider the following preference lists for four men and women. Apply a "man proposing" Gale-Shapley algorithm to find a stable matching.

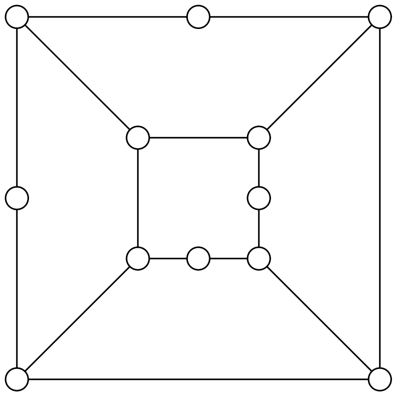
m_1	m_2	m_3	m_4
w_1	w_4	w_4	w_4
w_3	w_3	w_3	w_2
w_2	w_1	w_1	w_1
w_4	w_2	w_2	w_3

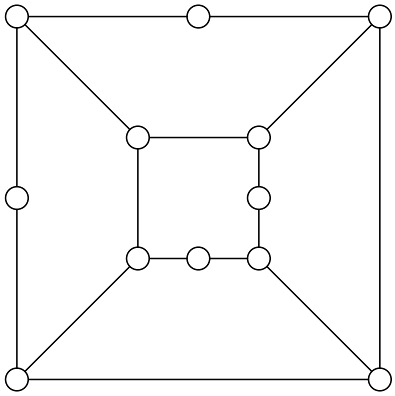
w_1	w_2	w_3	w_4
m_2	m_1	m_1	m_1
m_3	m_2	m_4	m_4
m_4	m_4	m_2	m_3
m_1	m_3	m_3	m_2

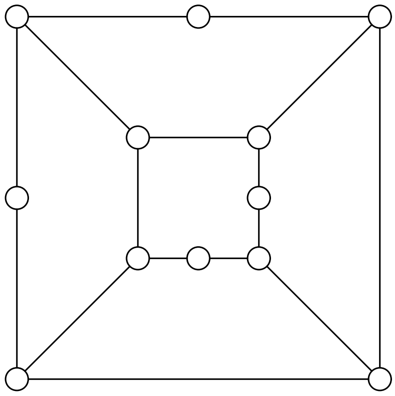
Points											
1a 3	2 7	3a 4	4a 7	5 3	6 7	7a 10	8 9	9 8	10 6	11 7	12 6
1b 6		3b 2	4b 1			7b 4					
9	7	6	8	3	7	14	9	8	6	7	6
Maximum score = 90											**Grade = 1 + score/10**

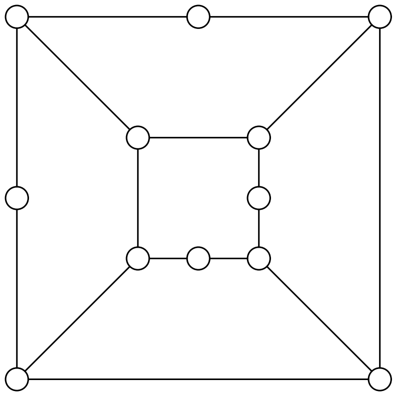
Name:

Problem 7b

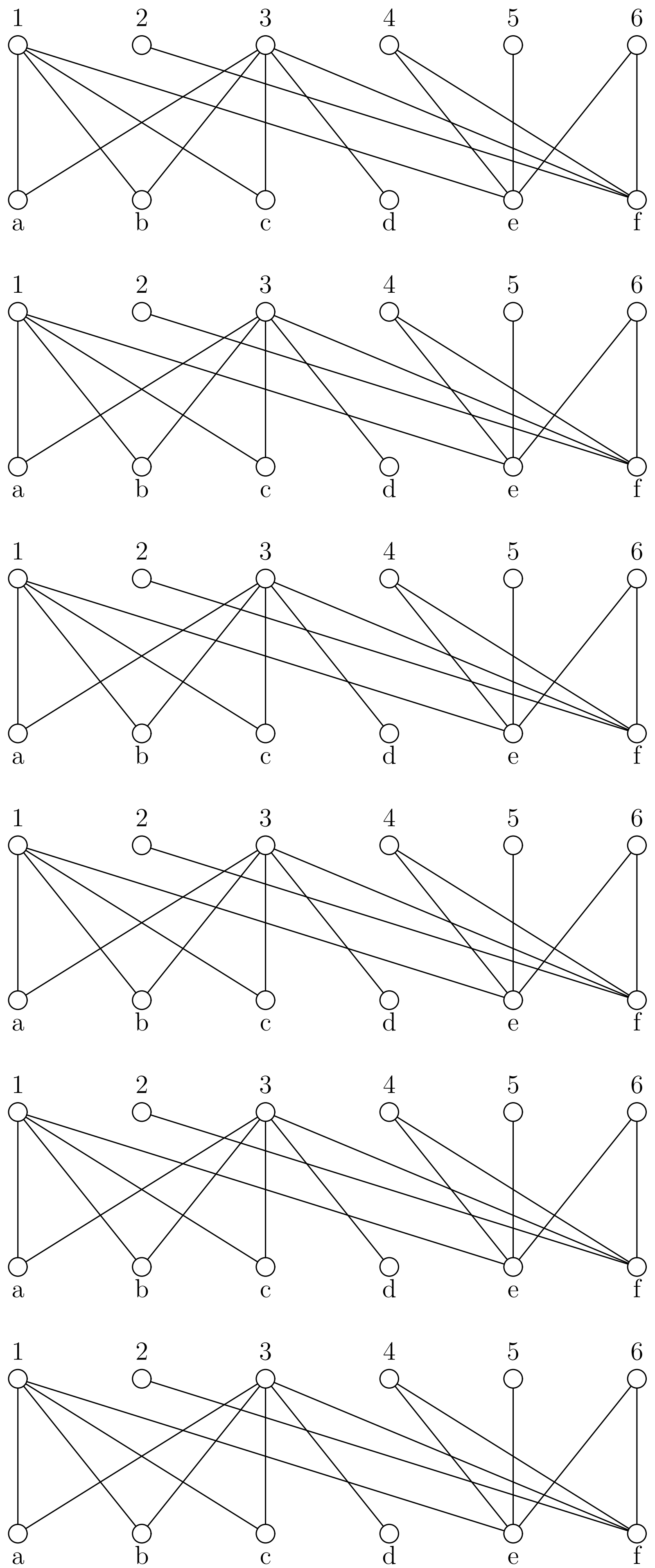








Problem 9


1 2 3 4 5 6
a b c d e f
1 2 3 4 5 6
a b c d e f
1 2 3 4 5 6
a b c d e f
1 2 3 4 5 6
a b c d e f
1 2 3 4 5 6
a b c d e f
1 2 3 4 5 6
a b c d e f