Faculty of Sciences VU University Amsterdam	Discrete Mathematics Exam 18:30-21:15 5-2-2014

(1) This exam consists of 12 problems.
(2) For problems 7b and 9 you may use the separate worksheet. **If you do this, write your name on it and hand it in.**
(3) Books, notes, calculators, etc., may not be used.
(4) Give explanations about and reasoning for your solutions.

Problem 1

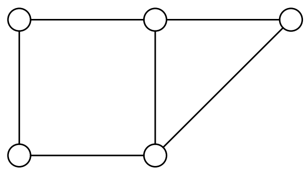
(a) How many different strings of letters can you form by rearranging the letters in the word "ABRACADABRA"? Give this number as an expression involving factorials and justify it, but do not evaluate it to an explicit number.
(b) How many 4-tuples (x_1, x_2, x_3, x_4) of non-negative integers are there that satisfy both $x_1 + x_2 + x_3 + x_4 = 8$ and $x_4 \neq 3$?

Problem 2

Let $N \geq 4$ be an integer. We randomly pick four *distinct* integers from $\{1, \ldots, N\}$ and compute their sum. Doing this 700 times we obtain 700 sums. Show that for $N \leq 62$, at least four of those 700 sums must have the same value.

Problem 3

Compute the chromatic polynomial of the following graph.



Problem 4

(a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} - 1$ $(n \geq 2)$ with $a_0 = 6$ and $a_1 = 11$.
(b) Use your answer to compute a_6.

Problem 5

The generating function $G(x)$ satisfies $2xG(x) = x^2G(x) - x$. Express the coefficient of x^k $(k \geq 0)$ in $G(x)$ in terms of k.

Problem 6

A zoo wants to put six lions $(a, \ldots, f$ for short) into five enclosures $(1, \ldots, 5$ for short). However, lions a and b cannot be put in the same enclosure, nor lions b and c, nor lions d and e. Give a explicit formula that computes the number of ways in which this can be done, but do *not* evaluate it to an explicit number.

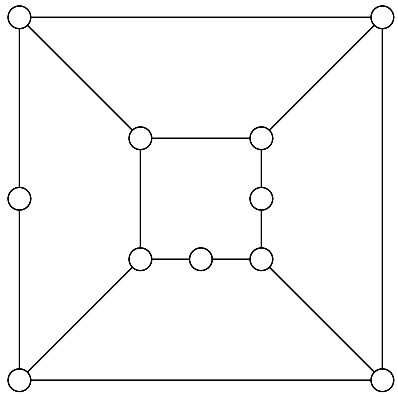
Problem 7

(a) Let G be a graph with 6 vertices and 11 edges.

(i) Show that G must be connected.

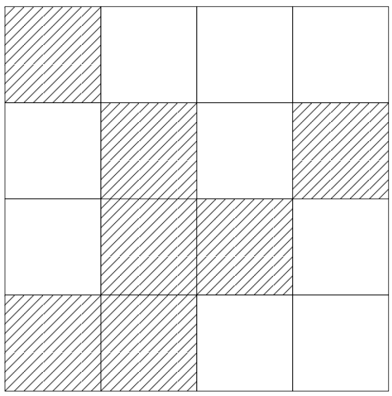
(ii) Give two examples of such a graph: one that admits an Eulerian closed chain, and one that does not admit any Eulerian chain, closed or not.

(b) Does the following graph have a Hamiltonian cycle? Either give such a cycle, or give an argument why it does not exist.



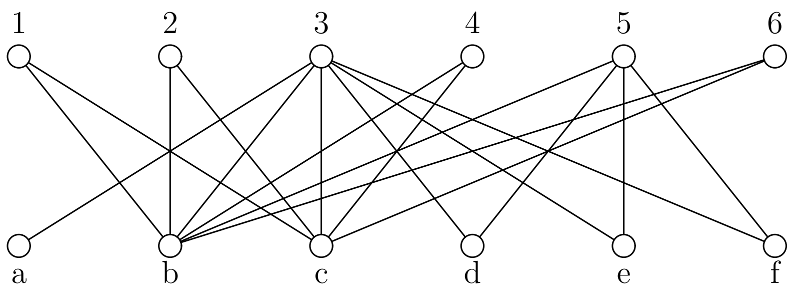
Problem 8

Use the reduction method discussed in class in order to compute the rook polynomial of the following board. (The board consists of the shaded squares.)



Problem 9

Let $G = (X, Y, E)$ with $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{a, b, c, d, e, f\}$ be the bipartite graph below. Starting with the matching M consisting of $\{2, b\}$, $\{3, c\}$ and $\{5, e\}$, use the algorithm discussed in class to find a maximum matching for G. What is $\delta(G)$? Also, find an explicit subset S of X that realizes $\delta(G)$, as well as a minimum covering for G.



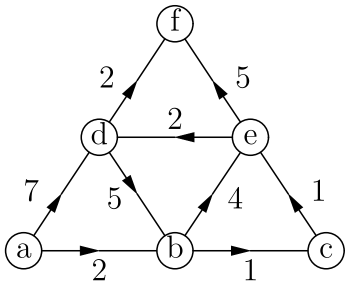
Problem 10

The following matrix gives the cost r_{ij} of worker i on job j. Use the Hungarian algorithm to find an optimum (i.e., with lowest total cost) job assignment.

$$\begin{pmatrix} 3 & 5 & 2 & 4 \\ 4 & 2 & 4 & 4 \\ 2 & 1 & 2 & 5 \\ 5 & 6 & 5 & 6 \end{pmatrix}$$

Problem 11

Use Dijkstra's algorithm to find a shortest path from vertex a to vertex f in the following directed network.



Problem 12

Consider the following preference lists for four men and women. Apply a "woman proposing" Gale-Shapley algorithm to find a stable matching.

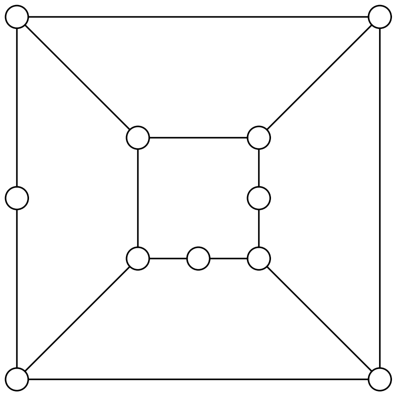
m_1	m_2	m_3	m_4
w_1	w_4	w_4	w_1
w_3	w_1	w_3	w_2
w_2	w_3	w_1	w_3
w_4	w_2	w_2	w_4

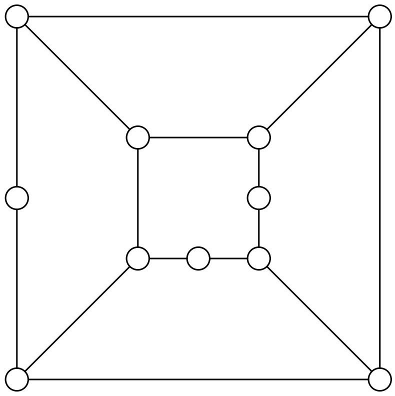
w_1	w_2	w_3	w_4
m_2	m_1	m_1	m_4
m_3	m_2	m_4	m_1
m_1	m_4	m_2	m_3
m_4	m_3	m_3	m_2

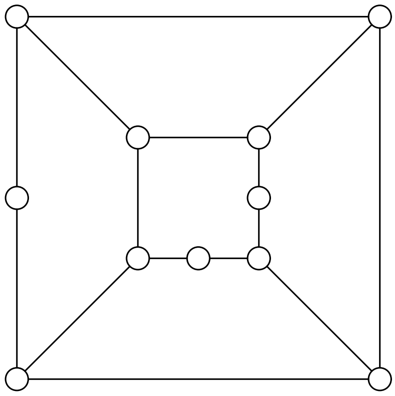
Points											
1a 3 1b 5	2 6	3 6	4a 7 4b 1	5 3	6 7	7a 12 7b 4	8 9	9 8	10 6	11 7	12 6
8	6	6	8	3	7	16	9	8	6	7	6
Maximum score = 90											**Grade = 1 + score/10**

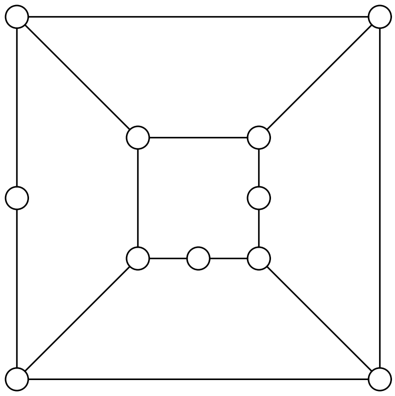
Name:

Problem 7b

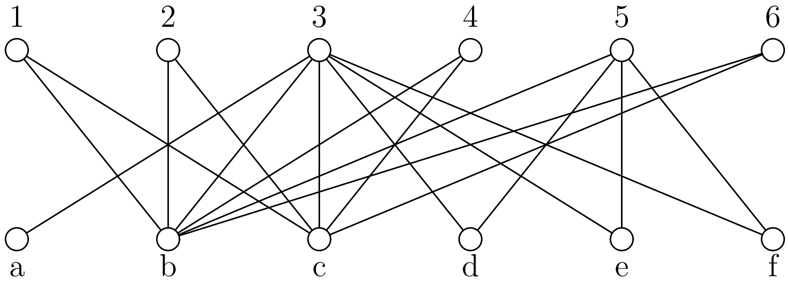


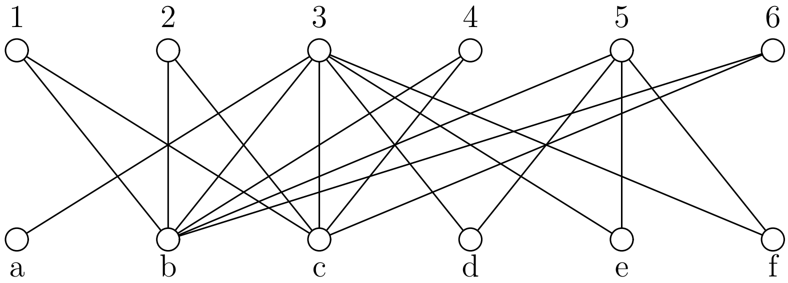






Problem 9


1 2 3 4 5 6
a b c d e f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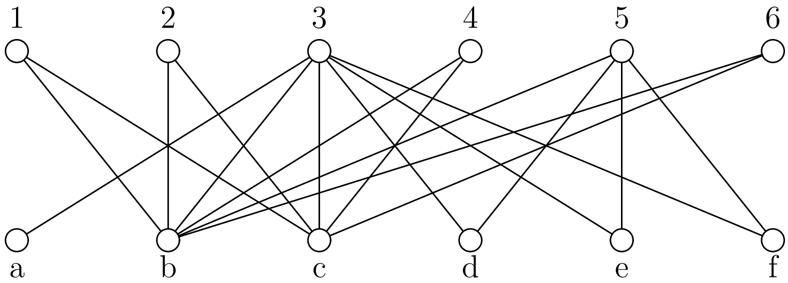
a b c d e f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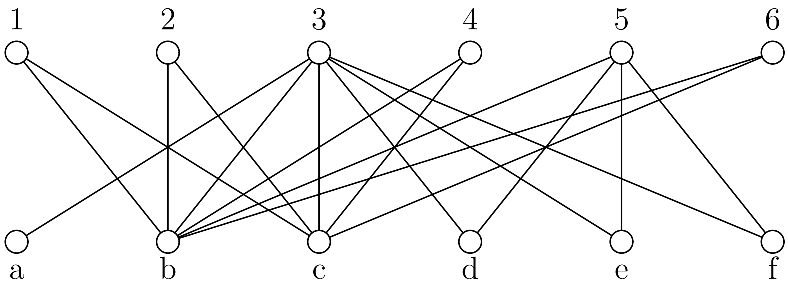
a b c d e f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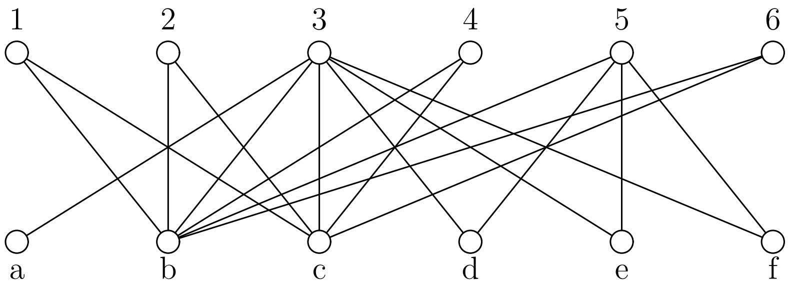
a b c d e f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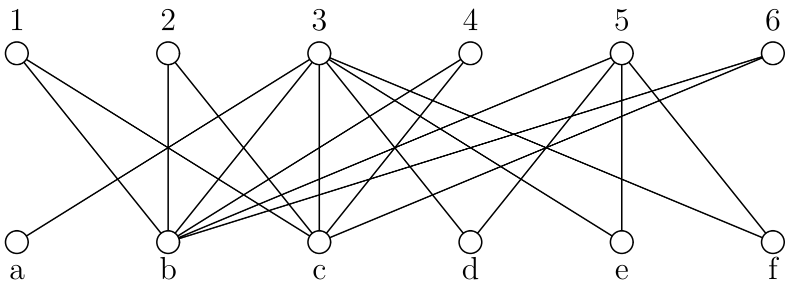
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