Faculty of Sciences	**Discrete Mathematics**
VU University Amsterdam	**2nd Partial exam 12:00-14:00 17-12-2013**

(1) This exam consists of 7 problems.
(2) For problems 2, 3 and 4 you may use the separate worksheet. **If you do this, write your name on it and hand it in.**
(3) Books, notes, calculators, etc., may not be used.
(4) Give explanations about and reasoning for your solutions.

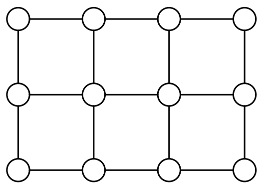
Problem 1

(a) How many of the integers 1,2,...,700 are not divisible by 4, 5 or 6?

(b) What is the number of strings of length 8 on the letters a, b, c, d, e, and f, in which neither the pattern $deaf$ nor the pattern $fade$ appears? Give an explicit formula that computes this number, but do not evaluate it to a number.

Problem 2

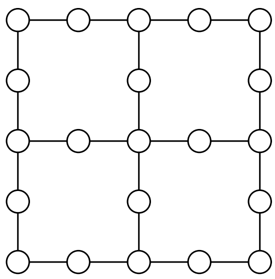
(a) Formulate a theorem that explains why the following graph does not have an Eulerian closed chain.



(b) Give a way of changing this graph into a multigraph that has an Eulerian closed chain, where you are only allowed to connect vertices that are already connected. What is the minimum number of new lines that are needed for this?

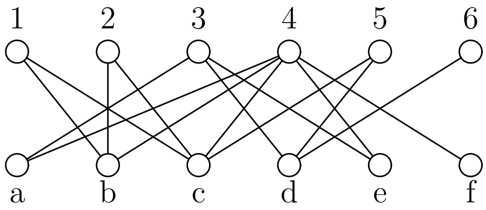
Problem 3

Does the following graph have a Hamiltonian circuit, or does it have a Hamiltonian chain? In each case justify your answer by either explaining why it does not exist, or exhibiting such a circuit or chain.



Problem 4

Let $G = (X, Y, E)$ with $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{a, b, c, d, e, f\}$ be the bipartite graph below. Starting with the matching M consisting of $\{2, b\}$, $\{3, d\}$ and $\{4, c\}$, use the algorithm discussed in class to find a maximum matching for G. What is $\delta(G)$? Also, find an explicit subset S of X that realizes $\delta(G)$, as well as a minimum covering for G.



Problem 5

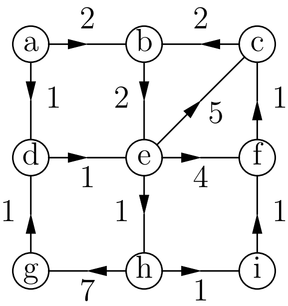
(a) The following matrix gives the cost r_{ij} of worker i on job j. Use the Hungarian algorithm to find an optimum (i.e., of lowest total cost) job assignment.

$$\begin{pmatrix} 8 & 5 & 5 & 6 \\ 7 & 17 & 11 & 13 \\ 10 & 4 & 3 & 6 \\ 4 & 13 & 10 & 18 \end{pmatrix}$$

(b) List all possible assignments of lowest total cost explicitly.

Problem 6

Use Dijkstra's algorithm to find a shortest path from vertex a to vertex c in the following directed network.



Problem 7

Consider the following preference lists for four men and women. Apply a "man proposing" Gale-Shapley algorithm to find a stable matching.

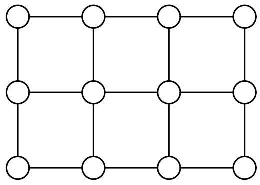
m_1	m_2	m_3	m_4
w_1	w_4	w_1	w_1
w_2	w_3	w_3	w_3
w_3	w_2	w_4	w_2
w_4	w_1	w_2	w_4

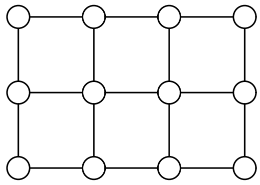
w_1	w_2	w_3	w_4
m_1	m_4	m_1	m_4
m_2	m_3	m_4	m_1
m_3	m_1	m_2	m_3
m_4	m_2	m_3	m_2

Points													
1a	8	2a	8	3	10	4	12	5a	10	6	12	7	8
1b	10	2b	8					5b	4				
	18		16		10		12		14		12		8
Max. score = 90							**Grade = 1 + score/10**						

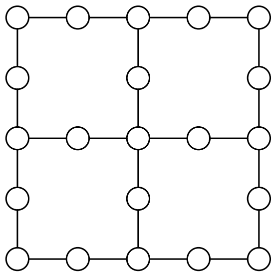
Name:

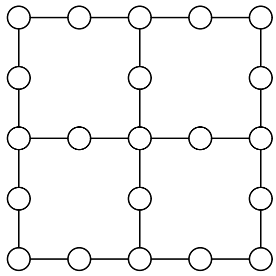
Problem 2





Problem 3





Problem 4