Hertentamen, Calculus I (Diff. & Int. I)
Dinsdag 7 januari, 18:30 – 21:15, 2014

Naam_____
Studentnummer_____

Puntentotaal = # pts + 10

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**ESSAY. Write your answer and WORK in the space provided.**

1) Find a suitable linear approximation that lets you estimate the value $1000^{1/5}$ and give the estimate. (6 pts) (Hint: $1024^{1/5} = 4$).

2) Solve the initial value problem $y^{(2)} + 2y^{(1)} + 2y = 0$; $y(0) = 4$, $y^{(1)}(0) = -2$. (6 pts)

3) Use information from the function and its first two derivatives to sketch the graph of the function

$f(x) = x\,e^{-x^2}$ (8 pts)

4) Find the second derivative of the function $f(x) = \sqrt{x^2 + 1}$. (6 pts)

5) Find a general solution to the first order linear differential equation $\frac{dy}{dx} - 2y - \frac{1}{1 + e^{-2x}} = 0$. (6 pts)

6) Evaluate the integral $\int \frac{49\,dx}{(x+4)^2\,(x-3)}$. (8 pts)

7) Evaluate the limit $\lim_{x \to 0} \left( \frac{1}{\ln(1 + x^2)} - \frac{1}{x^2} \right)$. (7 pts)

8) If $f^{(1)}(x) = 0$ throughout an interval [a, b], prove that f(x) is constant on the interval. (8 pts)

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. (5 pts)**

9) The equations $x = 2$, $x = 4$, $y = 1/x$, and $y = 0$ define the bounds of a region of the plane. Find the volume of the solid obtained by rotating the region about the x–axis. 9) _____

A) $\frac{3\pi}{4}$ cubic units

B) $\frac{\pi}{3}$ cubic units

C) $\frac{\pi}{2}$ cubic units

D) $\frac{\pi}{4}$ cubic units

E) $\frac{2\pi}{3}$ cubic units

10) Find the Taylor polynomial of degree 4 for the natural logarithm function $\ln x$ about $x = 2$. 10) _____

A) $\ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 - \frac{1}{64}(x-2)^4$

B) $1 + \frac{1}{2}(x-2) + \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 + \frac{1}{64}(x-2)^4$

C) $\ln 2 + \frac{1}{2}(x-2) + \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 + \frac{1}{64}(x-2)^4$

D) $\ln 2 + \frac{1}{2}(x+2) - \frac{1}{8}(x+2)^2 + \frac{1}{24}(x+2)^3 - \frac{1}{64}(x+2)^4$

E) $1 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 - \frac{1}{64}(x-2)^4$

11) Evaluate the integral $\int \frac{3}{x \ln x}\,dx$. 11) _____

A) $\ln|\ln x| + C$

B) $3\ln|\ln x| + C$

C) $\frac{3}{\ln|\ln x|} + C$

D) $\frac{-3}{(\ln x)^2} + C$

E) $\ln(3|\ln x|) + C$

12) The population of a city is growing exponentially, increasing at the rate of 1.6% per year. A new bicycle route must be in place before the population doubles from its current value. How long (rounded up to the nearest year) does the city have before the new bicycle route must be completed? 12) _____

A) 44 years B) 40 years C) 50 years D) 55 years E) 60 years

13) Find the inverse of the following function: 13) _____

$f(x) = (x-3)^2$, if $x \geq 3$.

A) $f^{-1}(x) = 2 + \sqrt{x}$

B) $f^{-1}(x) = \sqrt{3+x}$

C) $f^{-1}(x) = 3 - \sqrt{x}$

D) $f^{-1}(x) = 3 + \sqrt{x}$

E) $f^{-1}(x) = 1 + \sqrt{x}$

14) In order to prove that $\lim_{x \to 2} x^2 = 4$ we need to find, for any given $\varepsilon > 0$, a corresponding 14) _____

number $\delta > 0$ such that if $0 < |x-2| < \delta$, then $|x^2 - 4| < \varepsilon$. Which of the following values of $\delta$ will do for a given $\varepsilon$ ?

A) $\delta = \min\{1, \varepsilon\}$

B) $\delta = \dfrac{\varepsilon}{4+\varepsilon}$

C) $\delta = \sqrt{\varepsilon + 4} - 2$

D) $\delta = \dfrac{\varepsilon}{4}$

E) $\delta = \dfrac{\varepsilon}{2}$

**TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false.**
**(5 pts)**

15) True or False: If $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} \left(f(x)\right)^{-3} = L^{-3}$. 15) _____