

Uitwerking 2^e deeltentamen (D2) Calculus (I)
(W/N/BWI/Ectr) 16/12/99

1. a) Van breuksplitsing toe:

$$\frac{-4}{(x+1)^2(x-1)} = \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{C}{x-1} = \frac{A(x-1) + B(x^2-1) + C(x+1)^2}{(x+1)^2(x-1)}$$

Dit geeft het stelsel vgl^t:

$$B + C = 0$$

$$A + 2C = 0$$

$$-A - B + C = -4$$

met oplossingen: $A = 2$, $B = 1$, $C = -1$ en
dan:

$$\int \frac{-4}{(x+1)^2(x-1)} dx = \int \frac{2}{(x+1)^2} dx + \int \frac{dx}{x+1} - \int \frac{dx}{x-1} =$$

$$= \frac{-2}{x+1} + \ln|x+1| - \ln|x-1| + C.$$

$$b) \int_1^{\alpha} \frac{\ln x}{x\sqrt{x}} dx \stackrel{\text{p.i.}}{=} \left[-\frac{2}{\sqrt{x}} \cdot \ln x \right]_1^{\alpha} - \int_1^{\alpha} -\frac{2}{\sqrt{x}} \cdot \frac{1}{x} dx =$$

$$= \left[-\frac{2 \ln \alpha}{\sqrt{\alpha}} - \frac{4}{\sqrt{\alpha}} \right]_1^{\alpha} = -\frac{2 \ln \alpha}{\sqrt{\alpha}} - \frac{4}{\sqrt{\alpha}} + 4 \rightarrow \boxed{4} \text{ als } \alpha \rightarrow \infty$$

2. a) Van het quotiënten criterium toe:

$$\left| \frac{u_{n+1}}{u_n} \right| = \frac{(n+1)!}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n!} = \frac{n+1}{e^{2n+1}} \rightarrow 0 < 1$$

als $n \rightarrow \infty$ dan reeks is (abs.) convergent

$$b) (i) \left| \frac{u_{n+1}}{u_n} \right| = \left| \frac{(3x)^{n+1}}{2^{n+2}} \cdot \frac{2^{n+1}}{(3x)^n} \right| = \left| \frac{3}{2} x \right|$$

dan conv. als $|x| < \frac{2}{3}$ en div. als

$$|x| > \frac{2}{3}. \text{ De conv. straal } R = \underline{\underline{\frac{2}{3}}}.$$