Uitwerking tentamen Calculus II — 12-11-1998

1 (i) Basis: $n=2$: $a_2=\sqrt{2+a_1}=\sqrt{2+7}=3<7=a_1$. Dus $a_2<a_1$.
Inductie: Neem aan dat voor zekere $m\in\mathbb{N}$, $m\geq 2$, geldt $a_m<a_{m-1}$.
We tonen nu aan dat dan volgt dat $a_{m+1}<a_m$. Welnu:
$a_{m+1}=\sqrt{2+a_m}\overset{!}{<}\sqrt{2+a_{m-1}}=a_m$ (vanwege monotonie van de wortelfunctie en het feit dat alle $\sqrt{\ }$ bestaan, omdat $a_m\geq 0$, zie (ii)). Dus $\forall n\in\mathbb{N}$ geldt $a_{n+1}<a_n$, dus (a_n) is dalend

(ii) We tonen aan dat $a_n\geq 0$ $\forall n$. Basis: $n=1$: $a_1=7\geq 0$.
Inductie: Neem aan dat voor zekere $m\in\mathbb{N}$ geldt dat $a_m\geq 0$.
We tonen aan dat dan ook $a_{m+1}\geq 0$. Welnu:
$a_{m+1}=\sqrt{2+a_m}\overset{!}{\geq}\sqrt{2}>0$. Dus $\forall n\in\mathbb{N}$: (a_n) naar beneden begrensd door 0.

(iii) Volgens de monotone convergentiestelling heeft (a_n) een limiet, zeg L. Dan geldt: $L=\sqrt{2+L}$, dus $L^2-L-2=0$, dus $L=2$, want $L=-1$ voldoet niet (zie (ii)).

2 (i) $\left|\dfrac{(i+2)^n}{\sqrt{n}}\right|=\dfrac{(\sqrt{5})^n}{\sqrt{n}}\longrightarrow\infty$, dus alg. term $\not\to 0$, dus reeks divergent

(ii) $\left|\dfrac{(-i)^n}{n^3}\right|=\dfrac{1}{n^3}$ en $\sum\frac{1}{n^3}$ is convergent, dus $\sum\frac{(-i)^n}{n^3}$ convergeert absoluut.

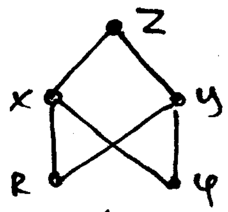
3 Homogeen: $y''+4y=0$. karakt. vgl: $\lambda^2+4=0$, $\lambda=\pm 2i$.
Dus $y_h=c_1\cos 2x+c_2\sin 2x$. Kies part. opl: $y_p=Ax+B$. Dan $y_p'=A$ en $y_p''=0$. Dus: $0+4(Ax+B)=4x$, dus $A=1$ en $B=0$: $y_p=x$
Alg opl: $y_{ALG}=c_1\cos 2x+c_2\sin 2x+x$.
Beginwaarden invullen: $y(0)=0$: $0=c_1$, dus $y_{ALG}=c_2\sin 2x+x$.
$y'(0)=7$: $7=2c_2+1$, dus $c_2=3$. Dus: $y=3\sin 2x+x$.

4 $\dfrac{\partial f}{\partial x}=4x-2y+2$ $\quad\dfrac{\partial f}{\partial y}=10y-2x-1$ $\quad\dfrac{\partial f}{\partial x}=\dfrac{\partial f}{\partial y}=0\Rightarrow(x,y)=(-\tfrac{1}{2},0)$

$\dfrac{\partial^2 f}{\partial x^2}=4$ $\quad\dfrac{\partial^2 f}{\partial y^2}=10$ $\quad\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}=-2$. Dus $D(-\tfrac{1}{2},0)=4\cdot 10-(-2)^2=36>0$

Dus (ook $f_{xx}>0$): er bevindt zich een minimum in $(-\tfrac{1}{2},0)$.

5



$$\frac{\partial z}{\partial R} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial R} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial R} = \frac{\partial z}{\partial x}\cdot\cos\varphi + \frac{\partial z}{\partial y}\cdot\sin\varphi \qquad \left|\ \times R\sin\varphi\right.$$

$$\frac{\partial z}{\partial \varphi} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \varphi} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \varphi} = \frac{\partial z}{\partial x}\cdot -R\sin\varphi + \frac{\partial z}{\partial y}\cdot R\cos\varphi \qquad \left|\ \times \cos\varphi\right.$$

$+$

$$R\sin\varphi\frac{\partial z}{\partial R} + \cos\varphi\frac{\partial z}{\partial \varphi} = (R\sin^2\varphi + R\cos^2\varphi)\frac{\partial z}{\partial y} = R\frac{\partial z}{\partial y}$$

Dus $\dfrac{\partial z}{\partial y} = \sin\varphi\dfrac{\partial z}{\partial R} + \dfrac{\cos\varphi}{R}\dfrac{\partial z}{\partial \varphi}$.

6

$$\int_\Omega xz\,dx\,dy\,dz = \left(\begin{array}{l}\text{Ga over op pool/cilinder coördinaten}\\ x = R\cos\varphi \quad y = R\sin\varphi \quad z = z\end{array}\right)$$

$$= \int_{\varphi=-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{R=0}^{1}\int_{z=0}^{1-R} R^2\cos\varphi\cdot z\,dz\,dR\,d\varphi = \int_{\varphi=-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{R=0}^{1} R^2\cos\varphi\cdot\tfrac{1}{2}(1-R)^2\,dR\,d\varphi =$$

$$= \int_{\varphi=-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\varphi\,d\varphi\cdot\int_{R=0}^{1}\tfrac{1}{2}(R^4-2R^3+R^2)\,dR = \sin\varphi\Big[_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cdot\tfrac{1}{2}\left(\tfrac{1}{5}R^5-\tfrac{2}{4}R^4+\tfrac{1}{3}R^3\right)\Big|_{R=0}^{1} =$$

$$= 2\cdot\tfrac{1}{2}\cdot\left(\tfrac{1}{5}-\tfrac{1}{2}+\tfrac{1}{3}\right) = \frac{1}{30}$$