Formuleblad Calculus voor Informatica

- **Exponenten:**

$$(ab)^r = a^r b^r$$
$$a^r a^s = a^{r+s}$$
$$(a^r)^s = a^{rs}$$
$$\frac{a^r}{a^s} = a^{r-s}$$

- **Trigonometrie:**

$$\sin^2 x + \cos^2 x = 1$$
$$\begin{aligned}\cos(2x) &= \cos^2 x - \sin^2 x\\ &= 1 - 2\sin^2 x\\ &= 2\cos^2 x - 1\end{aligned}$$
$$\sin(2x) = 2\sin x \cos x$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

- **Limieten:**

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$
$$\lim_{x\to\infty} e^x = \infty$$
$$\lim_{x\to\infty} e^{-x} = 0$$
$$\lim_{x\to\infty} \ln x = \infty$$
$$\lim_{x\to 0+} \ln x = -\infty$$
$$\lim_{x\to 0} \frac{\ln(x+1)}{x} = 1$$
$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$
$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$$
$$\lim_{n\to+\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$
$$\lim_{x\to+\infty} \frac{e^x}{x^k} = \infty, \text{ voor vaste } k \in \mathbb{R}$$
$$\lim_{x\to 0+} x^k \ln x = 0, \text{ voor vaste } k > 0$$
$$\lim_{x\to\infty} \frac{\ln x}{x^k} = 0, \text{ voor vaste } k > 0$$

- **Rekenregels differentiëren:**

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$
$$\frac{d}{dx}(cf(x)) = cf'(x)$$
$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$
$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

- **Afgeleiden:**

$$\frac{d}{dx} x^k = kx^{k-1}$$
$$\frac{d}{dx} \sin x = \cos x$$
$$\frac{d}{dx} \cos x = -\sin x$$
$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$
$$\frac{d}{dx} e^x = e^x$$
$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$
$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$
$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

- **Rekenregels integreren:**

$$\int af(x) + bg(x)\,dx$$
$$= a\int f(x)\,dx + b\int g(x)\,dx$$
$$\int f'(g(x))g'(x)\,dx = f(g(x)) + C$$
$$\int f'(x)g(x)\,dx$$
$$= f(x)g(x) - \int f(x)g'(x)\,dx$$
$$\int_a^b f'(x)\,dx = f(b) - f(a)$$
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

- **Primitieven:**

$$\int x^k\,dx = \frac{1}{k+1}x^{k+1} + C,$$
$$(k \neq -1)$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$
$$\int \sin x\,dx = -\cos x + C$$
$$\int \cos x\,dx = \sin x + C$$
$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C$$
$$\int e^x\,dx = e^x + C$$
$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$
$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

- **Reeksen:**

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \ |x| < 1$$
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$
$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$
$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$
$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},$$
$$|x| < 1$$

- **Complexe getallen:** In het vervolg is $z = x + iy$, $z \in \mathbb{C}, x, y \in \mathbb{R}$.
Zij $a \in \mathbb{C}$. Dan is $|z - a|$ de afstand van a tot z in het complexe vlak (Gauss–vlak).

$$i^2 = -1$$
$\bar{z} = x - iy$, de geconjugeerde van z
$$|z| = \sqrt{z \cdot \bar{z}} = \sqrt{x^2 + y^2}$$
$$|z^n| = |z|^n$$
$$\arg(z^n) = n\arg(z) \pmod{2\pi}$$
$$e^{i\phi} = \cos\phi + i\sin\phi$$
$$e^{\pi i} = -1$$