Faculteit der Exacte Wetenschappen	tentamen Calculus 2
Afdeling Wiskunde	22-12-2006
Vrije Universiteit	08:45-10:45 uur

1. Bereken:

   a) $\displaystyle\int_0^{\pi/2} \sin x\sqrt{\cos x}\, dx,$

   b) $\displaystyle\int_{-1}^{0} \frac{4x+1}{x^2+2x+2}\, dx,$

   c) $\displaystyle\int_2^{\infty} \frac{1}{x^2-1}\, dx.$

2. Onderzoek de volgende reeksen op absolute convergentie, dan wel relatieve convergentie, dan wel divergentie.

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+2^n},$ b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}\ln n},$ c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\sqrt{1+\frac{1}{n^2}}.$

3. Gegeven is de machtreeks

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n5^n}.$$

   a) Bereken de convergentiestraal R van deze machtreeks.

   b) Onderzoek het gedrag van de machtreeks in de randpunten $x = R$ en $x = -R$ van het convergentie-interval.

4. Bepaal de machtreeksontwikkeling van de functie $f(x) = \dfrac{x}{2-3x}$ en bepaal het interval waarop deze machtreeks convergeert.

Z.O.Z.

5. Bepaal zowel $p_2(x)$, het Taylorpolynoom van de orde 2, als $R_2(x)$, de restterm, van de functie $f(x) = \sqrt{4x+1}$ rond het punt $x = 2$.

6. De functie $f : \mathbb{R}^2 \to \mathbb{R}$ is gegeven door

$$f(x,y) = 2x^2 + 4xy - xy^2 - 2.$$

   a) Bepaal de vergelijking van het raakvlak aan de grafiek van f in het punt $(1,1,3)$.
   b) Bepaal de stationaire punten (critical points) van f.
   c) Ga na in welke van de bij b) gevonden punten f een lokaal extreme waarde aanneemt. Geef aan of het om een lokaal maximum of minimum gaat.

7. a) Bereken, door verwisseling van integratievolgorde:

$$\int_0^4 \int_{\sqrt{x}}^2 \cos(y^3)\, dy\, dx.$$

   b) Het gebied $S \subset \mathbb{R}^2$ wordt gegeven door:

$$S := \left\{ (x,y) \in \mathbb{R}^2 \,\middle|\, y \geq x \text{ én } x^2 + y^2 \leq 4 \right\}.$$

   Bereken:

$$\int\int_S \sqrt{x^2+y^2}\, dA.$$

Normering:

1 : a) 2	2 : 7	3 : a) 3	4 : 4	5 : 4	6 : a) 3	7 : a) 3
b) 3		b) 3			b) 3	b) 3
c) 4					c) 3	
9	7	6	4	4	9	6

$$\text{Eindcijfer} = \frac{\#\ \text{punten}}{5} + 1$$

Summary of Convergence Tests

NAME	STATEMENT	COMMENTS
Divergence Test (10.5.1)	If $\lim_{k\to+\infty} u_k \neq 0$, then $\sum u_k$ diverges.	If $\lim_{k\to+\infty} u_k = 0$, then $\sum u_k$ may or may not converge.
Integral Test (10.5.4)	Let $\sum u_k$ be a series with positive terms, and let $f(x)$ be the function that results when k is replaced by x in the general term of the series. If f is decreasing and continuous for $x \geq a$, then $\sum_{k=1}^{\infty} u_k$ and $\int_a^{+\infty} f(x)\,dx$ both converge or both diverge.	This test only applies to series that have positive terms. Try this test when $f(x)$ is easy to integrate.
Comparison Test (10.6.1)	Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be series with nonnegative terms such that $a_1 \leq b_1,\ a_2 \leq b_2, \ldots, a_k \leq b_k, \ldots$ If $\sum b_k$ converges, then $\sum a_k$ converges, and if $\sum a_k$ diverges, then $\sum b_k$ diverges.	This test only applies to series with nonnegative terms. Try this test as a last resort; other tests are often easier to apply.
Limit Comparison Test (10.6.4)	Let $\sum a_k$ and $\sum b_k$ be series with positive terms such that $\rho = \lim_{k\to+\infty} \frac{a_k}{b_k}$ If $0 < \rho < +\infty$, then both series converge or both diverge.	This is easier to apply than the comparison test, but still requires some skill in choosing the series $\sum b_k$ for comparison.
Ratio Test (10.6.5)	Let $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to+\infty} \frac{u_{k+1}}{u_k}$ (a) Series converges if $\rho < 1$. (b) Series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.	Try this test when u_k involves factorials or kth powers.
Root Test (10.6.6)	Let $\sum u_k$ be a series with positive terms such that $\rho = \lim_{k\to+\infty} \sqrt[k]{u_k}$ (a) The series converges if $\rho < 1$. (b) The series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.	Try this test when u_k involves kth powers.
Alternating Series Test (10.7.1)	If $a_k > 0$ for $k = 1, 2, 3, \ldots$, then the series $a_1 - a_2 + a_3 - a_4 + \cdots$ $-a_1 + a_2 - a_3 + a_4 - \cdots$ converge if the following conditions hold: (a) $a_1 \geq a_2 \geq a_3 \geq \cdots$ (b) $\lim_{k\to+\infty} a_k = 0$	This test applies only to alternating series.
Ratio Test for Absolute Convergence (10.7.5)	Let $\sum u_k$ be a series with nonzero terms such that $\rho = \lim_{k\to+\infty} \frac{\lvert u_{k+1}\rvert}{\lvert u_k\rvert}$ (a) The series converges absolutely if $\rho < 1$. (b) The series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.	The series need not have positive terms and need not be alternating to use this test.

Determining Convergence