

1) a) $\begin{bmatrix} 0 & 2 & -3 & 1 & 2 \\ 1 & 0 & 2 & -1 & -1 \\ 3 & -1 & 0 & 1 & 11 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & -3 & 1 & 2 \\ 0 & -1 & -6 & 1 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & -3 & 1 & 2 \\ 0 & -1 & -6 & 1 & 14 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & -3 & 1 & 2 \\ 0 & -1 & -6 & 1 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 6 & -1 & -14 \\ 0 & 2 & -3 & 1 & 2 \end{bmatrix}$ $C_1 = 3$
 $C_2 = -2$; $P = 3a - 2b - 2c$
 $C_3 = -2$

b) Am letzten konvergenz matrix by a is the 2 in last alle in spile leaf; das {a, b, c} wurde ein basis

c) $\cos \alpha = \frac{\vec{a} \cdot \vec{c}}{\|\vec{a}\| \cdot \|\vec{c}\|} = \frac{-6}{\sqrt{5} \sqrt{13}} = \frac{-6}{\sqrt{65}} \approx -0,74$; $\alpha \approx 138,1^\circ$

d) $\vec{a} - k\vec{p} \perp \vec{p}$; $\vec{a} \cdot \vec{p} - k\vec{p} \cdot \vec{p} = 0$
 $k = \frac{\vec{a} \cdot \vec{p}}{\vec{p} \cdot \vec{p}} = \frac{32}{4+1+121} = \frac{32}{126} = \frac{16}{63}$

2) a) $\sum_{n=0}^{\infty} 5 \cdot 3^{1-n} = 15 + 5 + \frac{5}{3} + \frac{5}{9} + \dots = \frac{15}{1 - \frac{1}{3}} = 22\frac{1}{2}$
 wenn MK mit $n = \frac{1}{3}$ ist unkonvergenz!

b) $n = \frac{1}{x} (x > 0)$; $-1 < \frac{1}{x} < 1$; $\text{dus } x > 1$
 $\text{Beim } \frac{a}{1-r} = \frac{5x}{1 - \frac{1}{x}} = \frac{5x^2}{x-1}$

c) $\frac{5x^2}{x-1} = 20$; $5x^2 = 20x - 20$; $x^2 - 4x + 4 = 0$; $(x-2)^2 = 0$; $x = 2$

3) a) $f'(x) = -2 \sin(2x) - 2 \cos x$; $f'(0) = -2$
 $f(0) = 1$; $K = k \cdot y$; $y = f(0) + f'(0) \cdot x = 1 - 2x$

b) $\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \dots$
 $2 \sin x = 2(x - \frac{x^3}{3!} + \dots)$

$1022x - 2 \sin x \approx 1 - 2x - 2x^3 + \frac{1}{3}x^3 + \frac{2}{3}x^5 + (-\dots)$
 ist grad, keine reihe.

4) a) $y' = \frac{M}{1 + k e^{-st}}$; $y(0) = 25$
 $M = \text{max} = 150$; $k = 3$

b) $y(13) = \frac{150}{1 + 3 e^{-13}} = 55,5$; $s = 0,01$

5) a) $\int_1^e \frac{x^2 - 8x + 16}{x^2} dx = \int_1^e (1 - \frac{8}{x} + 16x^{-2}) dx = [x - 8 \ln x - \frac{16}{x}]_1^e = (e - 8 + 16) - (1 - 8 + 16) = e - 16 - 9 \approx 10,3$

b) $\int_1^e x^{\frac{3}{2}} \ln x dx = \frac{2}{7} x^{\frac{3}{2}} \ln x - \int_1^e \frac{2}{7} x^{\frac{3}{2}} \cdot \frac{1}{x} dx = \frac{2}{7} x^{\frac{3}{2}} \ln x - (\frac{2}{7}) x^{\frac{3}{2}} \Big|_1^e = \frac{2}{7} e^{\frac{3}{2}} - \frac{2}{7} e^{\frac{3}{2}} - (0 - \frac{2}{7}) = \frac{2}{7} \approx 0,28$

c) $g(x) = \int_0^x (e^{1+x} - (-2)x) dx = \int_0^x (e^{1+x} + 2x) dx = [e^{1+x} + x^2]_{-1}^0 = (e + 1) - (e^{-1} - 1) = e - \frac{1}{e} + 2 \approx 1,2$

d) a) $\frac{\partial^2 z}{\partial x^2} = -1$; $\text{grad}(z) = (-1, 2)$; $\tan \varphi = |(-1, 2)| = \sqrt{5}$; $\varphi \approx 66^\circ$

b) Hesses in richtung $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ ist $(1, 2) \cdot (\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}) = \frac{2-1}{\sqrt{5}} = 1$; $p = \frac{3}{5}$

f) $f(x, y) = x^2 y + y^2 - y$; $f_x(x, y) = 2xy$; $f_y(x, y) = x^2 + 2y - 1$
 $f_x(0, 1) = 0$; $f_y(0, 1) = 0 + 2 - 1 = 1$

a) $f(0, 1) = 0 + 1 - 1 = 0$; $f_x(0, 1) = 0$; $f_y(0, 1) = 1$
 $\text{rank Hess } H = -3 + 0 - 2(1-1) = -3 - 2y + 2 = -1 - 2y$; $\Delta = f_{xx} f_{yy} - f_{xy}^2$

b) $f_{xx}(0, 1) = 0$; $f_{yy}(0, 1) = 2$; $f_{xy}(0, 1) = 0$; $\Delta = 0 \cdot 2 - 0^2 = 0$
 ist kein Minimum

c) $\int_0^1 \int_0^2 (2x - y) dx dy = \int_0^1 (x^2 - yx) \Big|_{x=0}^{x=2} dy = \int_0^1 (4 - 2y) dy = [4y - y^2]_0^1 = 3$

d) $\int_1^2 \int_0^2 \sin(x^2) dx dy = \int_1^2 x \sin(x^2) dx = -\frac{1}{2} \cos(x^2) \Big|_1^2 = -\frac{1}{2} \cos 4 + \frac{1}{2} \cos 1$

