Basiswiskunde wortel I 8-4-99

1) a) $\begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 2 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$

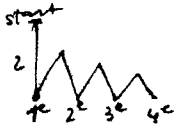
b) $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 1 & 1 & 2 & 4 \\ 1 & -1 & 5 & 12 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 2 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $\begin{array}{l} x_3 = 2 \\ x_2 = -1 \\ x_1 = 1 \end{array}$

2) a) $\sqrt{1+9+4+1} = \sqrt{15}$ b) $\cos\alpha = \dfrac{a \cdot b}{\|a\| \cdot \|b\|} = \dfrac{1+5-4+3}{\sqrt{15} \cdot \sqrt{15}} = 0{,}2$; $\alpha \approx 78°$.

c) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & \\ 3 & 1 & 7 & \\ -2 & 2 & -18 & \\ 1 & 3 & -11 & \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & \\ 0 & -2 & 10 & \\ 0 & 4 & -20 & \\ 0 & 2 & -10 & \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & \\ 0 & 1 & -5 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{array}\right]$ Dus: $\underline{c} = 4\underline{a} - 5\underline{b}$.

3) a) MR met $r = x^2 - 1$; conv. als $-1 < x^2 - 1 < 1$; $0 < x^2 < 2$
Dus $-\sqrt{2} < x < \sqrt{2}$ en $x \neq 0$.

b) $\text{Som} = \dfrac{t_1}{1-r} = \dfrac{1}{1-(x^2-1)} = \dfrac{1}{2-x^2} = \dfrac{4}{7}$; $8 - 4x^2 = 7$; $4x^2 = 1$; $x = \pm\frac{1}{2}$.

c) 

$\text{weg} = \underset{1^e \text{ landing}}{2} + \underset{2^e \text{ landing}}{2 * 0{,}8 * 2} + \underset{3^e}{2 * 0{,}8^2 * 2} + \cdots + \underset{10^e}{2 * 0{,}8^9 * 2}$

$= 2\left[1 + 2 * 0{,}8 + 2 * 0{,}8^2 + \cdots + 2 * 0{,}8^9\right] =$

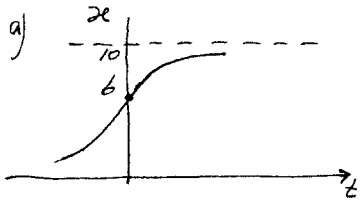
$= 2 \cdot \dfrac{1 - 0{,}8^{10}}{1 - 0{,}8} = 10\,(1 - 0{,}8^{10}) \approx 8{,}9 \text{ m}.$

(ii) $2 * \dfrac{1}{1-0{,}8} = 10$ m.

4) a) $a = 5$; $b = e^{-0{,}3} \approx 0{,}74$ b) $N'(t) = 5 \cdot e^{-0{,}3t} \cdot (-0{,}3) = -1{,}5e^{-0{,}3t} = -9$

$t \approx \dfrac{\ln 6}{-0{,}3} \approx -5{,}97.$

c) $\ln N = \ln 5 - 0{,}3t$; $0{,}3t = \ln\left(\frac{5}{N}\right)$; $t = 3\frac{1}{3} * \ln\left(\frac{5}{N}\right)$

5) a) 

$10 - \dfrac{30}{3 + 2e^{-0{,}3t}} < \dfrac{1}{10}$

$30 + 20e^{-0{,}3t} - 30 < \dfrac{3 + 2e^{-0{,}3t}}{10}$

$198\, e^{-0{,}3t} < 3$; $t > 13{,}97$

b) $H = \dfrac{30}{3 + 2e^{-0{,}3t}}$; $3H + 2He^{-0{,}3t} = 30$

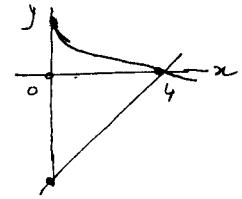
$\dfrac{30 - 3H}{H} = 2e^{-0{,}3t}$

$\ln\left(\dfrac{30 - 3H}{H}\right) = \ln 2 - 0{,}3t$.

Dus: $p = -0{,}3$
$q = \ln 2 \approx 0{,}69$.

6) a) $\int_4^5 \dfrac{x^2 + x - 12}{2x+1}\,dx = \ln|x^2 + x - 12|\Big]_4^5 = \ln 18 - \ln 8 = \ln 2\frac{1}{4} \approx 0{,}81$

b) $\int_0^{\pi} x \sin 2x\,dx \underset{(p.i.)}{=} -\frac{1}{2}x\cos 2x\Big]_0^{\pi} - \int_0^{\pi} -\frac{1}{2}\cos 2x\,dx = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\Big]_0^{\pi} = -\frac{1}{2}\pi$

c) 

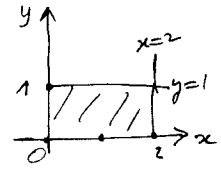
$\int_0^4 ((2 - \sqrt{x}) - (x-4))\,dx = \left(6x - \frac{2}{3}x^{3/2} - \frac{1}{2}x^2\right)\Big]_0^4 = 10\frac{2}{3}$

7) $f(x,y) = 2x^2 + 3xy - y^2$

a) $f_x(x,y) = 4x + 3y$ $\quad f_x(1,2) = 4 + 6 = 10$
$f_y(x,y) = 3x - 2y$ $\quad f_y(1,2) = 3 - 4 = -1$
grootste stijging in richting gradiënt: $\binom{10}{-1}$.

b) $z = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$
$= 4 + 10(x-1) + (-1)\cdot(y-2) = 10x - y - 4$

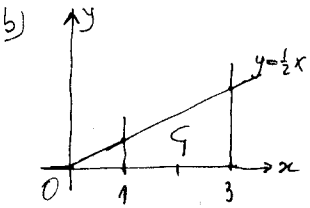
c) Niveau is 4; richting niveaulijn $\perp$ gradiënt, dus $\binom{1}{10}$.

8) a) 

$\int_0^2 \left(\int_0^1 (y - 2x)\,dy\right) dx =$

$\int_2 \left[\frac{1}{2}y^2 - 2xy\right]_{y=0}^{y=1} = \frac{1}{2} - 2x$

$\int_0^2 \left(\frac{1}{2} - 2x\right) dx = \frac{1}{2}x - x^2\Big]_0^2 = 1 - 4 - 0 = -3$

b) 

$\int_1^3 \left(\int_0^{\frac{1}{2}x} e^{x^2}\,dy\right) dx =$

$\int_0^{\frac{1}{2}x} e^{x^2}\,dy = y e^{x^2}\Big]_{y=0}^{y=\frac{1}{2}x} = \frac{1}{2}x e^{x^2}$

$\int_1^3 \frac{1}{2}x e^{x^2}\,dx = \frac{1}{4}e^{x^2}\Big]_1^3 = \dfrac{e^9 - e}{4}$.