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Let $X_1, X_2, \ldots$ be independent and normally distributed random variables with density p_{θ_0}, where θ_0 denotes the 'true parameter'. Define, for $\theta \in \mathbb{R}$, the function

$$\psi_\theta(x) = (x - \theta)^3 \quad (x \in \mathbb{R})$$

and consider the Z-estimator $\hat{\theta}_n$ defined as a zero of the function

$$\Psi_n(\theta) = \mathbb{P}_n \psi_\theta = \frac{1}{n} \sum_{i=1}^{n} \psi_\theta(X_i)$$

a) Using properties of the function Ψ_n, show that $\hat{\theta}_n$ is well defined. In other words: show that Ψ_n has exactly one point where it becomes zero.

b) Prove that $\hat{\theta}_n \to^P \mu$.

You may use that $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^6 e^{-x^2/2}\, dx = 15$.

c) The random variables $\sqrt{n}(\hat{\theta}_n - \theta_0)$ are asymptotically normally distributed. Use the parameter you expect for the asymptotic variance to compute the asymptotic relative efficiency of $\hat{\theta}_n$ and the sample mean $\bar{X}_n$

5. a) Given a sample $X_1, \ldots, X_n$ from a distribution with probability density f, give the definition of the kernel estimator $\hat{f}_{n,h}$ based on this sample and show that the Mean Squared Error (MSE) of $\hat{f}_{n,h}(x)$ decomposes in a bias- and variance term.

b) Formulate the Glivenko Cantelli theorem and give an outline of its proof (line of thought, basic ingredients).

1a	1b	1c	2	3a	3b	4a	4b	4c	5a	5b
4	3	3	3	4	3	2	2	3	3	2

The final grade is (number of points + 4)/3.6

Good luck with the exam!