Exam Applied Stochastic Modeling
13 February 2012

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give 2.25 points when correctly answered.
The use of a calculator is allowed. A table with the Poisson distribution is attached.

1. Consider the $M|G|1|1$ queue, thus a single server with no waiting places. The service times are *hypoexponential* with 2 phases, which is the sum of 2 exponential distributions with different rates μ_1 and μ_2.
a. Formulate the PASTA principle.
b. Use renewal theory to calculate the probability that an arrival is rejected.
c. Model this system as a continuous-time Markov chain and calculate its stationary distribution. Use two states to model the service time. Verify that the answers correspond to what is found under b.
d. Draw the state-transition diagram of the $M|G|1$ queue with the same service-time distribution.

2. Consider the $M|G|1$ queue with the service time distribution S hypoexponential as defined in problem 1. The parameters are as follows: $\lambda = 1$, $\mu_1 = 2$, and $\mu_2 = 3$.
a. Calculate $\mathbb{E}S$ and $\sigma^2(S)$.
b. Calculate $\mathbb{E}W_Q$ and $\mathbb{E}W$.
c. Formulate Little's law and calculate $\mathbb{E}L_Q$ and $\mathbb{E}L$.

3. Consider the Erlang B system with exponential service times, thus the $M|M|s|s$ queue, with the usual parameters λ, μ, and s. Let π be the stationary distribution. Let N be a Poisson distributed r.v. with parameter λ/μ.
a. Show that $\pi(i) = \mathbb{P}(N = i)/\mathbb{P}(N \leq s)$, for $0 \leq i \leq s$.
b. Use this to calculate $\pi(0)$ and $\pi(s)$ for $\lambda = 5$, $\mu = 1$, and $s = 7$.
c. Calculate for the Erlang B system the long-run distribution α as seen by customers who enter the system.
d. Calculate $\alpha(0)$ and $\alpha(s)$ for $\lambda = 5$, $\mu = 1$, and $s = 7$.

4. Consider a continuous-review continuous-product deterministic inventory model with holding costs, order costs and 0 lead time (the EOQ model).
a. Give the optimal order size Q^* which minimizes the sum of holding and ordering costs.
We change the model as follows. When Q is ordered then a random amount is delivered which is normally distributed with expectation Q and some fixed standard deviation σ such that in all realistic cases a negative order size is very unlikely to occur.
b. Calculate the average holding costs for fixed Q.
c. Calculate the optimal order quantity.