Exam Applied Stochastic Modeling
17 December 2007

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 2, and 4 each give 2 points when correctly answered, question 3 can give 3 points.
The use of a calculator is allowed.
A table of the Poisson distribution is attached.

1. Consider a single-order inventory problem (the newsvendor problem), Poisson-distributed demand with average μ, costs for lost sales q, and costs for left-over items h.
a. Compute the optimal order level for $\mu = 5$, $q = 2$ and $h = 1$. Do the same for $\mu = 10$, $q = 2$ and $h = 1$.
b. Compare both answers and explain the findings.
c. Assume that we order thinking that $\mu = 5$, but actually $\mu = 10$. Compute the expected relative increase in costs compared to ordering when knowing that $\mu = 10$.

2. A queueing network consists of two queues each with a single server. New customers arrive to queue 1 according to a Poisson process with rate λ. After being served they move to queue 2. With probability p they leave the system after being served at queue 2; with probability $1-p$ they go back to queue 1, then go to queue 2, etc. The service times at both queues are exponential with rate μ_i for queue i.
a. Formulate the routing equations for this system. For which values of λ is the system stable?
b. Give the steady state equations of this system, formulated in term of λ, p, μ_1, and μ_2.

3. An entrepreneur has to accept or reject project offers. Offers arrive according to a Poisson process; he can only accept an offer when he is not currently working on a project. Projects have a stochastic duration S. Revenue is calculated per hour. The rate with which revenue is earned changes from project to project and has distribution R. R and S are independent.
a. Suppose that our entrepreneur accepts every offer when not working on a project. Give an expression for his long-run average revenue.
b. Calculate this expression for $\lambda = 1$, $\mathbb{E}S = 1$, and R uniform on $[0, 1]$.
c. Now suppose that the entrepreneur only accepts offers when $R \geq r$. Calculate the long-run average revenue for $r = 0.25$, 0.5, and 0.75. What is the value of r that maximizes the long-run average revenue?
d. Now suppose that the entrepreneur not only decides on the basis of R, but that he also knows the value of S. Does this information help him make better decisions? Motivate your answer.

4. Consider the $M|M|3|3$ queue.
a. Draw the state-transition diagram.
b. Give its stationary distribution.
c. Give the distribution as it is perceived by an arbitrary arriving customer. Motivate your answer.
d. Give the distribution as it is perceived by an arbitrary departing customer.

Table with values of P(X>k) with X a Poisson distributed random variable with mean mu

	values of mu									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	$\boxed{0.238}$	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	$\boxed{0.303}$
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000