Exam Applied Stochastic Modeling
7 March 2007

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 2, 3, and 4 each give 2 points when correctly answered, question 1 can give 3 points.
The use of a calculator is allowed.

1. Consider the $M|M|2|2$ queue.
a. Draw the state-transition diagram.
b. Give its stationary distribution.
c. Give the distribution as it is perceived by an arbitrary arriving customer. Motivate your answer.
d. Give the distribution as it is perceived by an arbitrary departing customer.

2. Consider a deterministic-demand continuous-review inventory model where we do not allow for backorders or lost sales.
a. Draw a typical time-inventory diagram and formulate the appropriate inventory model to compute the optimal order size.
b. Compute the optimal order size and the time between two orders for the following parameter values: order costs $K = 200$, holding costs $h = 1$ per day, and demand $\lambda = 4$ per day.
c. Now suppose orders are placed (if necessary) only on Monday, at noon. What is the optimal order size and the time between two orders?

3. Consider and $M|G|1$ queue with $\lambda = 1$ and the following service time distribution:

$$S = \begin{cases} 0 & \text{with probability } \frac{1}{2}, \\ 1 & \text{with probability } \frac{1}{2}. \end{cases}$$

a. Calculate $\mathbb{E}W$ and $\mathbb{E}W_Q$.
b. Do the same for

$$S = \begin{cases} 0 & \text{with probability } 1 - \frac{1}{2c}, \\ c & \text{with probability } \frac{1}{2c}. \end{cases}$$

c. Call the answers found under b $\mathbb{E}W(c)$ and $\mathbb{E}W_Q(c)$. Calculate $\lim_{c\to\infty} \mathbb{E}W(c)$ and explain intuitively the answer.

4. Consider an inhomogeneous Poisson process $N(t)$ on $[0, \infty)$ with rate function $\lambda(t) = t$.
a. Give an expression for $\mathbb{P}(N(t) = k)$.
b. Let X_1 be the moment of the first arrival. Give an expression for $\mathbb{P}(X_1 \leq t)$.
c. Calculate $\mathbb{E}X_1$.