Exam Applied Stochastic Modeling
18 December 2006, duration: 3 hours

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 3, and 4 each give 2 points when correctly answered, question 2 can give 3 points.
The use of a calculator is allowed.
A table of the Poisson distribution is attached.

1. Consider the $M|M|2$ queue.
a. Draw the state-transition diagram.
b. Give its stationary distribution.
c. Give the distribution as it is perceived by an arbitrary arriving customer. Motivate your answer.
d. Give the distribution as it is perceived by an arbitrary departing customer.

2. A multi-order deterministic-demand inventory model is considered with $K = 400$, $h = 1$, and $\lambda = 2$.
a. Calculate the optimal order quantity and the time between two orders.
b. One considers to set the re-order period to 28 time units. Estimate the consequences to the total costs compared to the optimal re-order period.
Now suppose the demand is Poisson distributed, with λ as expectation. The lead time is 2 time units, and the order quantity is 50.
c. At what inventory level should be ordered as to have less than 5% backorders?
d. Suppose we order at the level found under c, but that λ is actually twice as high. What would be the backorder percentage?

3. Consider the $M|G|1$ queue with $\lambda = 1$ and $\beta = 0.8$.
a. For exponential service times, calculate the expected waiting time.
~~b. For deterministic service times, calculate the expected waiting time.~~
c. Consider a queueing system with the queues of a and b in tandem. Which order (exponential first or deterministic first) will have a lower total waiting time? Motivate your answer.

4. Consider a homogeneous Poisson process on $[0, T]$ with rate λ.
a. What is the expected number of arrivals in this interval?
b. Let the first arrival occur at t. Conditioned on this event, what is the expected number of arrivals in $[0, T]$?
c. Calculate the expected number of arrivals in $[0, T]$ again, but now using the law of total probability and the answer found under b.
d. Repeat a, b and c for an inhomogeneous Poisson process.

Table with value of P(X>k) with X with a Poisson distributed random variable with mean mu

values of k	values of mu 1	2	3	4	5	6	7	8	9	10
0	0,632	0,865	0,950	0,982	0,993	0,998	0,999	1,000	1,000	1,000
1	0,264	0,594	0,801	0,908	0,960	0,983	0,993	0,997	0,999	1,000
2	0,080	0,323	0,577	0,762	0,875	0,938	0,970	0,986	0,994	0,997
3	0,019	0,143	0,353	0,567	0,735	0,849	0,918	0,958	0,979	0,990
4	0,004	0,053	0,185	0,371	0,560	0,715	0,827	0,900	0,945	0,971
5	0,001	0,017	0,084	0,215	0,384	0,554	0,699	0,809	0,884	0,933
6	0,000	0,005	0,034	0,111	0,238	0,394	0,550	0,687	0,793	0,870
7	0,000	0,001	0,012	0,051	0,133	0,256	0,401	0,547	0,676	0,780
8	0,000	0,000	0,004	0,021	0,068	0,153	0,271	0,407	0,544	0,667
9	0,000	0,000	0,001	0,008	0,032	0,084	0,170	0,283	0,413	0,542
10	0,000	0,000	0,000	0,003	0,014	0,043	0,099	0,184	0,294	0,417
11	0,000	0,000	0,000	0,001	0,005	0,020	0,053	0,112	0,197	0,303
12	0,000	0,000	0,000	0,000	0,002	0,009	0,027	0,064	0,124	0,208
13	0,000	0,000	0,000	0,000	0,001	0,004	0,013	0,034	0,074	0,136
14	0,000	0,000	0,000	0,000	0,000	0,001	0,006	0,017	0,041	0,083
15	0,000	0,000	0,000	0,000	0,000	0,001	0,002	0,008	0,022	0,049
16	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,004	0,011	0,027
17	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,002	0,005	0,014
18	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,002	0,007
19	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,003
20	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,002
21	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001
22	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
23	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
24	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
25	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000