Exam Applied Stochastic Modeling
15 February 2006, duration: 3 hours

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Question 1 can give 3 points, questions 2, 3 and 4 can give 2 points.
The use of a calculator and a dictionary are allowed.
Tables of the Poisson distribution and the standard normal distribution are attached.

1. Consider a system with 3 machines and 2 repairmen. Machines fail independently with rate λ. Each repairman repairs machines at rate μ. When one machine is down then only one repairman can work, when 2 or more machines are down both work.
a. Model this system as a birth-death process.
b. Calculate the stationary distribution and use this to derive the long-run expected number of machines that are functioning.
d. Derive the long-run distribution at moments that a machine fails.
e. Use this to derive the distribution of the long-run average time a machine waits before it is taken into service and calculate its expectation.

2. Consider a continuous-time multi-order deterministic-demand continuous-product inventory model with $\lambda = 5$, $K = 10$, $h = 1$ and $L = 1$.
a. Compute the optimal re-order level and re-order size.
Now demand is stochastic; it occurs according to a Poisson process with rate $\lambda = 5$. For the rest the system is the same. Items that are not available are backordered.
b. We use the same re-order policy. Estimate the probability that backorders occur in a cycle.
c. It is the objective to avoid backorders in at least 9 out of 10 cycles. How should we choose the re-order policy to achieve this?

3. The numbers of arrivals to a service center are noted during 100 days. The number at day i is given by x_i. We have $\sum_{i=1}^{100} x_i = 1763$ and $\sum_{i=1}^{100}(x_i - 17.63)^2 = 2351$.
a. Give a 95% confidence interval for the expected number of arrivals on a day.
b. Somebody thinks that the number of arrivals per day has a Poisson distribution. Is this likely to be the case? Motivate your answer.

4. Consider an $M|G|1$ queue with arrival rate 0.5 and service time distribution $S = X + Y$, with X and Y both exponentially distributed with rates 1 and 2, respectively.
a. Calculate $\mathbb{E}S$, $\mathbb{E}S^2$, $\sigma^2(S)$ and $c^2(S)$.
b. Calculate the expected waiting time and the expected sojourn time for the $M|G|1$ queue.
c. What is the probability that an arbitrary arrival finds an empty system?

Table with value of P(X>k) with X with a Poisson distributed random variable with mean mu

values of k	values of mu 1	2	3	4	5	6	7
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table with value of $P(0<X<x+y)$ with X a random variable with a standard normal distribution

values of x \ values of y	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499