Exam Applied Stochastic Modeling
20 December 2005, duration: 3 hours

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 2, and 4 each give 2 points when correctly answered, question 2 can give 3 points.
The use of a calculator is allowed.
A table of the standard normal distribution is attached.

1. Consider a service center with an infinite number of servers. Customers arrive according to a Poisson process with rate λ. The service time distribution S has distribution function F and expectation β.
a. What is the limiting distribution of the number of customers in the system? And what is its expectation?
b. Give an expression for the expected number of customers that are older than t time units.
c. What is the distribution of the number of customers that are older than t time units? Motivate your answer.

2. Consider a system of two parallel M/D/1 queues. Both have load 80%, but one has service times of length 1 and the other of length 10.
a. Calculate the system times in both systems, and the expected overall system time.
The manager of the system considers merging both queues to obtain economies of scale. We approximate the resulting M/D/2 queue by a single M/D/1 queue with double service speed. Customer are treated in the order of arrival. b. Caracterize the arrival process and the service time distribution of the resulting queueing system.
c. Calculate the system time in this new M/D/1 queue.
d. Compare the results found under a and c and give an intuitive explanation. How would you redesign the system as to obtain the lowest possible average system time?

3. Passengers arrive according to a Poisson process at a bus stop. Buses also arrive according to a Poisson process. All waiting passengers leave with the next bus.
a. What is the expected number of passengers waiting at the arrival of an arbitrary bus?
b. What is the distribution of the number of waiting passengers?
c. There are n seats in a bus and they arrive empty at the bus stop. What is the long-run proportion of passengers that do not have a seat?

4. Consider a periodic inventory model where the lead time is equal to the length of the period: an order is placed the moment the previous order arrives. The order policy is as follows: if, after the order arrival, the inventory level is x then an order is placed of $S - x$ items. Unmet demand is backordered. The demand in each period is i.i.d. normal(μ,σ^2) distributed.
a. Give the relation between the order levels of the successive re-order moments.
b. Give a formula for the probability that backorders occur in a certain interval and approximate the average inventory level.
c. Compute these numbers for $S = 10$, $\mu = 4$, and $\sigma^2 = 8$.

Table with value of $P(0<X<x+y)$ with X a random variable with a standard normal distribution

values of x \ values of y	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499