1. a. Let A be a closed subspace of a Lindelöf space X. Prove that A is Lindelöf.

   b. Let A be an uncountable subset of a Lindelöf space X. Prove that A has a limit point.

2. Let X denote the real line with the half-open interval topology, and let $Y = X \times X$.

   a. Prove that $\mathcal{B} = \{[a,b) \times [c,d) : a,b,c,d \in \mathbb{R}, a < b, c < d\}$ is a basis for Y.

   b. Show that Y is separable.

   c. Show that the line $y = -x + 1$ is a closed and discrete subspace of Y.

   d. Show that Y is not Lindelöf.

3. a. Prove that a finite product of discrete spaces is discrete.

   b. For $i \in \mathbb{N}$, let X_i be a discrete space containing more than one point. Prove that $\prod_{i=1}^{\infty} X_i$ is not discrete.

4. a. Let $f\colon X \to Y$ be a quotient map with Y connected and $f^{-1}(y)$ connected for every $y \in Y$. Prove that X is connected.

   b. Let (X,d) be a metric space with disjoint, non-empty, closed subsets A and B. Define $f\colon X \to [0,1]$ by:

$$f(x) = \frac{d(x,A)}{d(x,A) + d(x,B)}.$$

   Prove that f is well-defined and continuous. Prove that X is normal.

EINDE