Trial exam SPF II

Question 1.

Suppose that the risk free rate is 2% per year and the average dividend yield on the AEX stock index is 5% per year.

a) The current index value is 494 Euros and the 6-months futures contract on this index is quoted at 503 Euros. What arbitrage opportunity does this create and how can you realize it?
b) A UK investor would like to enter a pound sterling-valued forward contract on this index, expiring in 6 months. Current exchange rate is 1.3, suppose again that interest rates in EU are 2% and interest rates in UK are 2.5%. What is the forward price of such a contract and how the seller would hedge this contract?

Question 2.

Prove the following bound for European call options on a stock without dividend, using portfolio argument:

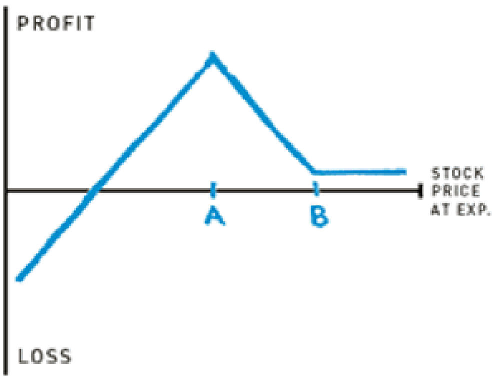
$$C_2 \leq \frac{E_3 - E_2}{E_3 - E_1} C_1 + \frac{E_2 - E_1}{E_3 - E_1} C_3,$$

where C_1, C_2 and C_3 are calls with the same expiry, T, and have exercise prices E_1, E_2 and E_3 respectively, where $E_1 < E_2 < E_3$.

Hint: Consider $E_2 = \lambda E_1 + (1 - \lambda) E_3$.

Question 3

Consider the payoff of a package:



Explain how to construct this payoff using regular puts. Explain what is your "bet" about the underlying stock price, if you purchase such a package and what is a likely current value of the stock price when this package is first purchased, in terms of A and B. What event will put you in a load of trouble, if you own such a package?

Question 4

Consider the following modification of a **binary option:** is a claim which pays 1 Euro if the stock price at maturity falls within some pre-specified interval: $a<S(T)<b$. Otherwise nothing will be paid out. Determine the arbitrage free price of such a binary option.
Hint: use the approach and the prices of usual binary options.

Question 5

a) LIBOR zero curve is flat at 1.5% (continuously compounded) out to 1.5 years. Swap rates for 2 and 3 year swaps (semi-annually paid) are 1.7% and 1.8% respectively. Estimate LIBOR zero rates for maturities 2, 2.5 and 3 years.

b) Under the terms of an interest rate swap, a bank has agreed to pay 1.5% per annum and receive 6-month LIBOR on a notional of 50M USD. Payments are done semi-annually. The swap has the remaining lifetime of 36 months. All the rates are as in a). The 6-month LIBOR at the last settlement date was 1.6% per annum. What is the value of the swap for the bank?

Question 6

Consider forward starting ATM option, so the option is sold at time *T(0)*, the strike is established at time *T(1)>T(0)* (e.g., for ATM option the strike will be *K=S(T(1)))* and the option's exercise time is *T(2)>T(1).*

Note that after time *T(1)*, forward starting puts and calls become regular puts and calls (as the strikes become fixed) and hence, regular put-call parity relationships will hold. So the below questions concern the period between *T(0)* and *T(1).*

a) Consider a very simple situation: underlying is a non-dividend paying stock and interest rates are zero. What is the put-call parity relation for forward starting ATM puts and calls?
b) Let's make the situation a bit more complicated: consider forward starting puts and calls which are now not ATM, but where the strike is going to be some multiplier *m* times the price at time *T(1): K=m*S(T(1))* (so at time *T(0)* we still do not know the strike but we will know it at time *T(1)*). What is now the parity relationship for forward starting puts and calls? Both dividends and interest rates are still zero.
c) How the put-call parity relationships you have found in a) and b) will change if the risk free interest rate is non-zero (but constant)?
d) How these relationships will change if the underlying stock pays dividends at the rate of *q*% p/a?

Question 7 (bonus)

Note: All equations and SDEs in this exercise are under risk-neutral measure Q.

Suppose the logarithm of the commodity price $X_t = lnS_t$ follows mean reverting process:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t.$$

A popular model for commodity's futures prices is similar to the usual buy-and-hold relationship between spot and forward prices ($F(t,T) = S_t e^{(r+c)(T-t)}$, where *c* is storage cost), but where the cost of carrying the commodity to expiration *(r+c)* is replaced by the so-called convenience yield *y(t)*:

$$F(t,T) = S_t e^{y(t)*(T-t)}. \qquad (1)$$

The convenience yield is assumed to be stochastic and is also governed by the mean reverting process with mean zero:

$$dy(t) = -ky(t)dt + sdZ_t\ .$$

Assume that the Brownian motions *W* and *Z* that drive the log-spot price and the convenience yield are independent.

a) Find the SDE for the so-called *aggregated convenience yield*:

$$Y(t,T) = y(t) * (T-t).$$

(so find the expression for $dY(t,T)$). Is it also a mean reverting process? What is its volatility?

b) Find the SDE for the logarithm of the futures price: $L(t,T) = lnF(t,T)$. Write that SDE in a usual form: $dL(t,T) = \mu dt + \tilde{\sigma} d\widetilde{W}_t$ and give expressions for μ and $\tilde{\sigma}$.
Hint: you can just take the logarithm of the relationship (1) and you do not even need to apply Ito's lemma.

c) Now find the SDE for the futures price: $dF(t,T)$. What is the volatility of $F(t,T)$?
Hint: here you will need Ito's lemma applied to the function $y = f(x) = e^x$, since $F(t,T) = e^{lnF(t,T)}$.

d) Describe how Black Scholes formula can be used to price futures options on $F(t,T)$ in this framework.
Hint: recall that you are operating under Q measure and, under this measure, futures prices are driftless martingales!